The Industrial development of Denmark 1840-1914

Niels Kœrgård
University of Copenhagen

1. Introduction

The date when the industrial breakthrough in Denmark took place has been discussed for a long time. The most specific theories from recent decades are those of Richard Willerslev (1954 and 1952), Svend Aage Hansen (1970) and Ole Hyldtoft (1984). Willerslev argues for an early revolution starting in the 1850s, while Hansen lays stress on the rapid growth in the 1890s. Hyldtoft tries to combine these two different views in a theory of long cycles. Hyldtoft does not see the cycles as substantial change in the aggregate growth rate, but mainly as a change in the composition of the factors of production. He divides the development into phases of capital deepening and of capital widening. In the periods 1840-1965 and 1896-1914 both the growth in the stock of capital and the rate of technological progress are very substantial, while the period 1865-1896 is characterized by a consolidation of already introduced techniques.

Some of the differences in attitude between the authors could be explained by the fact that Willerslev and Hyldtoft are historians, while Hansen is an economist. Hansen bases his theory mainly on figures for the aggregate national product, while Willerslev and Hyldtoft prefer more disaggregated series. They are especially interested in employment and in the amount of mechanical power in the manufacturing industries in Copenhagen (Hyldtoft's book deals with industrial growth in Copenhagen 1840-1914).

In this paper the industrial growth in Denmark is analyzed by means of methods utilised in mathematical statistics — methods which have not been used at all in the debate.

Two sections concentrate on the development in mechanical power. Section 2 tests whether changes in the characteristics really took place between the chosen periods and tries to estimate an optimal division in periods. Section 3 tries to explain the development in mechanical power by a logistic curve modelling the introduction of steam engines. In section 4 a number of other indicators are introduced, and a common trend is estimated by use of the methods of principal components. The paper ends with a short conclusion.

2. The different phases in the development

There are several different series for the development of mechanical power, but none of them is complete for the total period 1840-1914. In fig. 1 the data are summarized. It is clear that a change in the growth rate in 1865 is not a very distinct event and that a similar change in 1896 is indistinct because of lacking data.

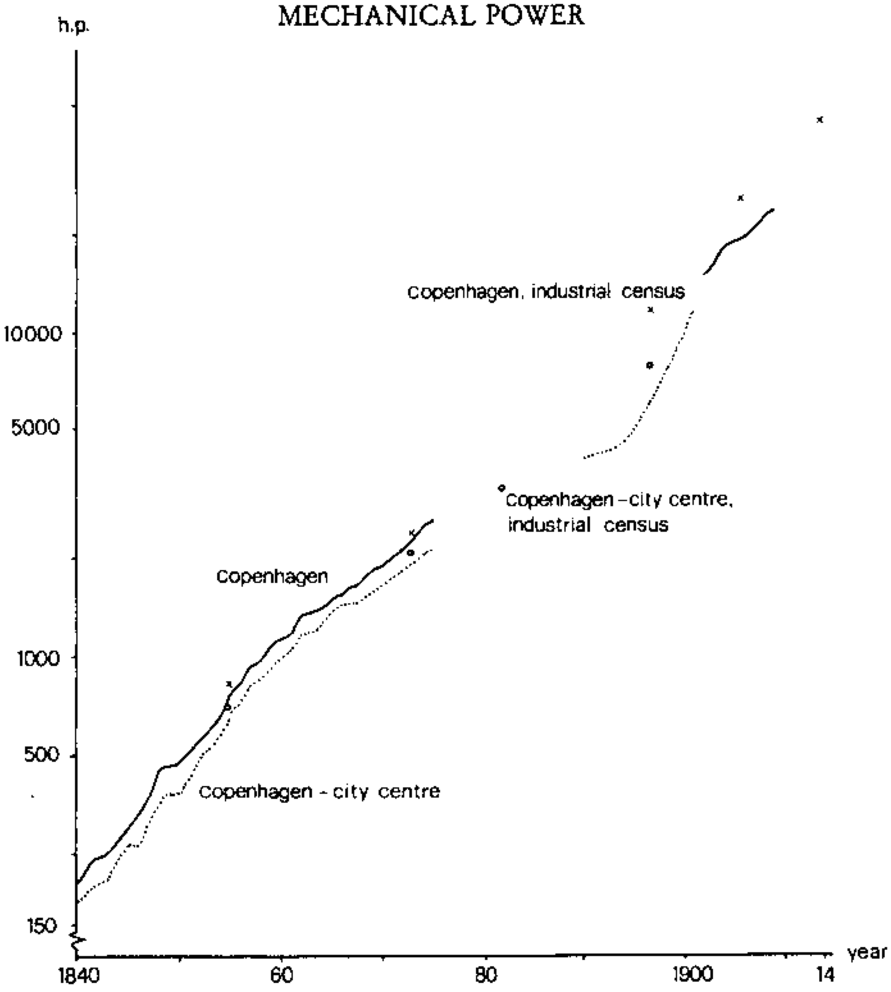
A more evident separation into three periods is seen in fig. 2 where mechanical power per employee is shown. The only extant data, however, are based on industrial censuses for eight different years for the period 1831-1914, and two of these (1831 and 1839) are previous to the period discussed in this paper.

A combination of the different series from fig. 1 could be used in connection with the broad definition of Copenhagen and the unbroken series from 1840-1875 and 1902-1909, supplemented by the industrial census data from 1897 and 1914 and adjusted down to the level of the other series by means of data from the years with both kinds of observations.[1]

---

[1] The relation between the two types of data is estimated for the years where both observations are available (ie. 1847, 1855, 1873, 1906). The industrial census data is

Fig. 1
MECHANICAL POWER



Source: Hyldtoft (1984) p. 54.

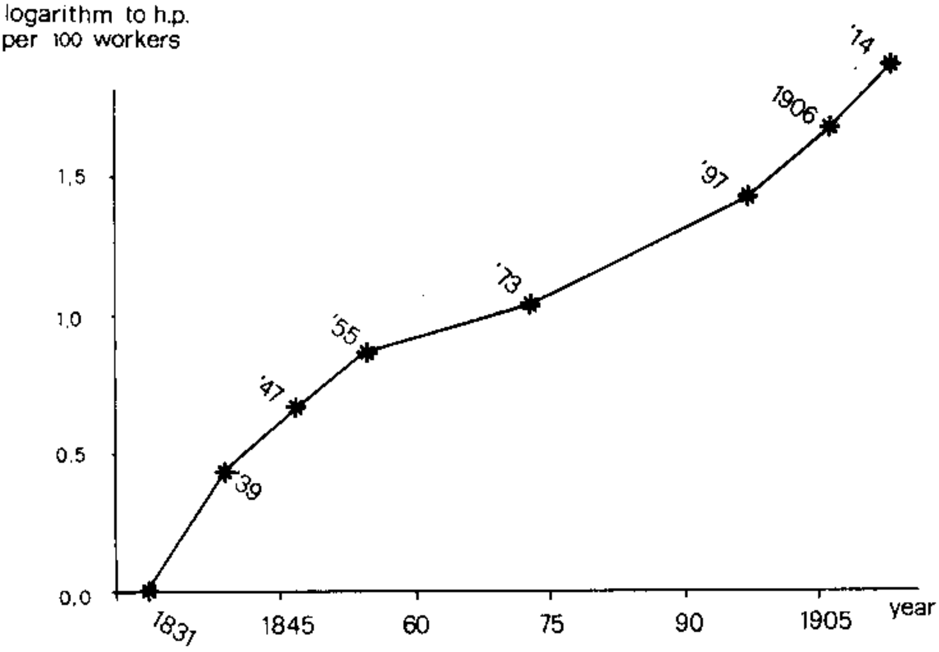
A log-linear trend is estimated for these data as shown in fig. 3.A. The estimated relation is:

$$\log HP_i = \underset{(1.6)}{-117.2} + \underset{(0.001)}{0.067}\, i \qquad (1)$$

$$R^2 = 0.99 \qquad D.W. = 0.11 \qquad N = 45$$

estimated to be 33% higher than the other data. The Industrial census data from 1897 and 1914 is then reduced by 33% and used together with the other data.

Fig. 2
THE DEVELOPMENT OF MECHANICAL
POWER FOR WORKERS 1830-1914



Source: Hyldtoft (1984) p. 52.

where log HP_i is the mechanical power extant in year i. The figures in brackets are the deviation of the coefficients, R^2 the degrees of explanation, D.W. is Durbin-Watson's test for autocorrelation and N is the number of observations.

It is seen that the log-linear trend is a rather exact description of the series — the degree of determination is about 99%, but D.W. indicates that there is a systematic deviation from the trend.

The problem is whether this systematic pattern of the deviation could be explained in the different phases for the three periods (1840-1865, 1865-1896, 1896-1914), as assumed by Ole

Hyldtoft. In fig. 3.B log-linear trends are estimated for the three periods separately. They are composed by a Chow-test for *switch in the relation.*[2] The test compares the sum of the squared residuals for the three trends, SSR_u, with the same sum when the three trends are restricted to be equal, SSR_R; it is the sum of squared residuals in fig. 3.B and fig. 3.A. The test-quantity is F-distributed. The actual test is:

$$\frac{(SSR_u - SSR_R) / (f\text{-}k)}{SSR_R / f} = \frac{(0.606 - 0.080)/4}{0.080/39} = 64.1 \quad (2)$$

where f is the number of degrees of freedom in the restricted estimation and k is the number of restrictions. It is seen that the test is extremely significant, and it could be concluded that Hyldtoft is right in assuming that there are different phases.

The next question is then whether 1865 and 1896 are the right demarcation points between the phases. Data for the years close to 1896 are so scarce that tests are impossible, and only the switch in 1865 should be tested. The unbroken series 1840-1875 are used as data and the model by R.E. Quandt for relations with shifting regimes is the statistical framework.

In this model (see Quandt, 1958 and Goldfeld & Quandt, 1976. An elementary introduction to the topic is found for instance in Johnston, 1984 chap. 10.2 and 10.4) the development is described with two different trends as in fig. 3.B. The model is:

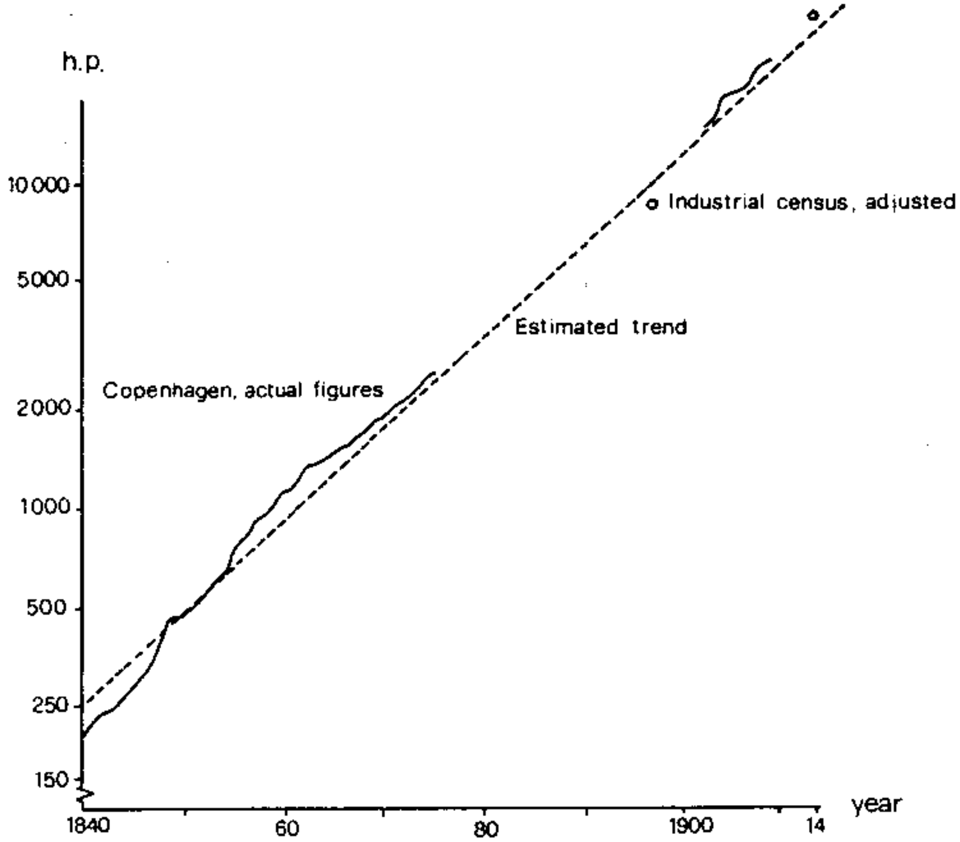
$$\begin{aligned} y_i &= \alpha_o + \alpha_1 i + u_{i1} \qquad \text{for } i \leqslant t \\ y_i &= \beta_o + \beta_1 i + u_{i2} \qquad \text{for } i > t \end{aligned} \quad (3)$$

where y_i is the mechanical power in year i, t the year for the switch, α and β are parameters measuring the two different trends and u_{i1} and u_{i2} are normal, independent distributed re-

---

[2] The Chow-test and other similar test are discussed in Brown, Durbin & Evans (1975), Farley, Hinick & McGuire (1975). An introduction to the test is in Johnston (1984 pp. 207-225) and an application for instance in Rasmussen & Kærgård (1980).

Fig. 3A
MECHANICAL POWER IN THE
INDUSTRY OF COPENHAGEN

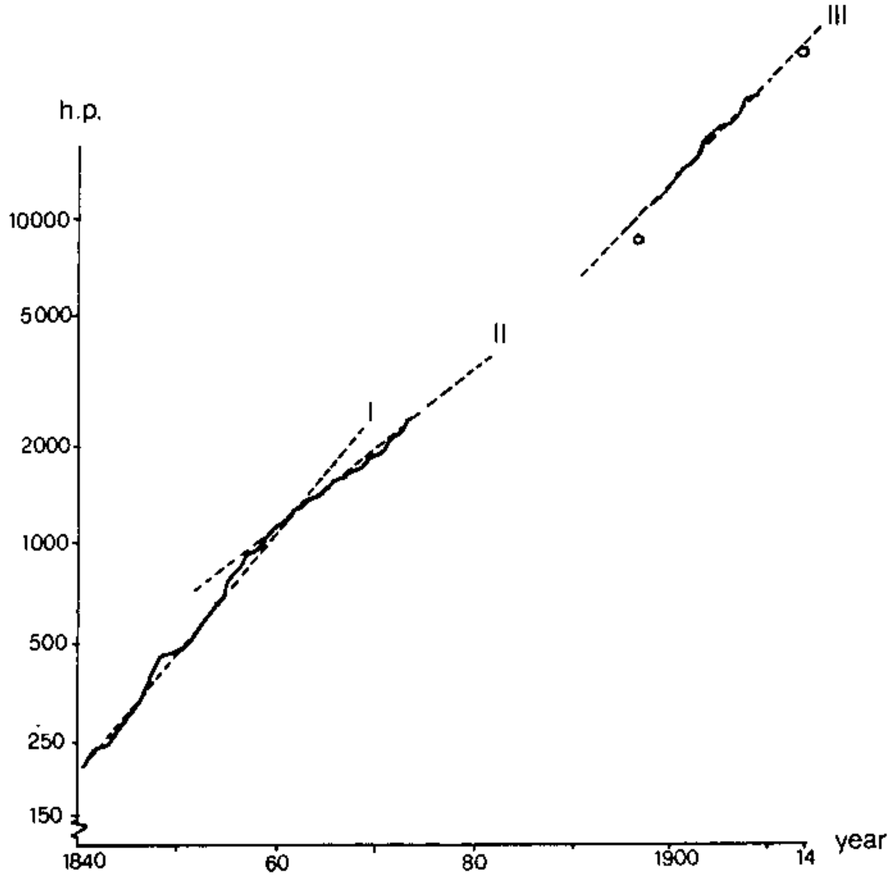


siduals with mean O and deviation σ_1, σ_2. There are then 7 parameters to estimate (α_0, α_1, β_0, β_1, σ_1, σ_2 and t).

The method developed by Quandt is a maximum-likelihood-method. The maximum-likelihood function is:

$$\ln L = -\frac{n}{2}\ln 2\pi - \frac{n}{2} - \frac{t}{2}\ln s_1^2 - \frac{n-t}{2}\ln s_2^2 \qquad (4)$$

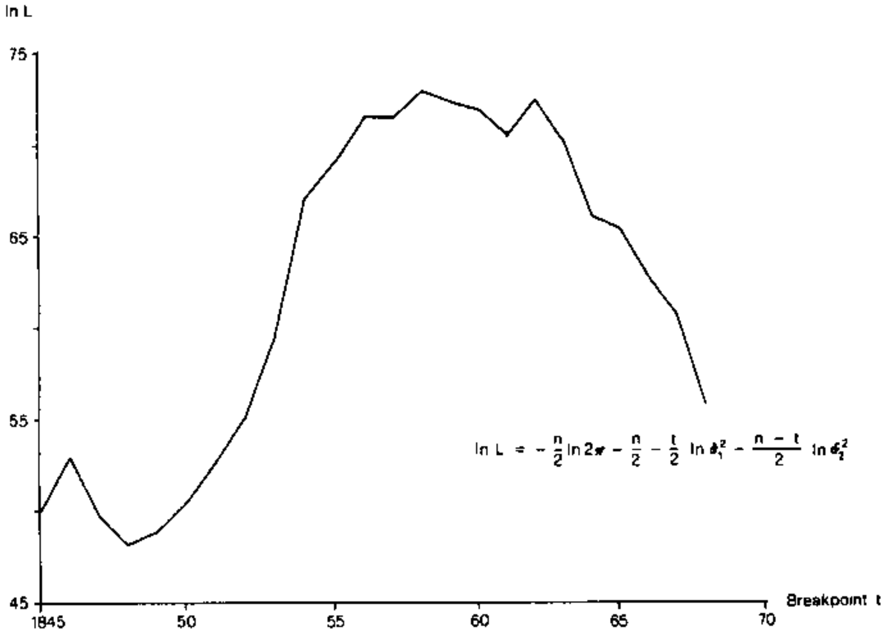
Fig. 3B



Source: See fig. 1.

where n is the total number of observation and s_1 and s_2 are maximum-likelihood-estimates of σ_1 and σ_2. If t is fixed, then 1n L could be calculated (s_1 and s_2 could be found by OLS for the two subsamples and (4) then calculated). The value of ln L is then calculated for different values of t, and ln L as a function of t is shown in fig. 4.

Fig. 4
THE LOG-LIKELIHOOD FUNCTION
FOR DIFFERENT BREAKPOINTS



It is seen that 1858 is the optimal estimate of t, but that the likelihood-function is rather flat from 1856 to 1862. The statistically optimal breakpoint is consequently about 7 years earlier than proposed by Ole Hyldtoft on the basis of more diffuse criteria, and the statistical analysis is strongly against an assumption placing the phase change after 1863.[3]

3. A theory of the development

In the last section it is concluded that the development of mechanical power in manufacturing industry in Copenhagen

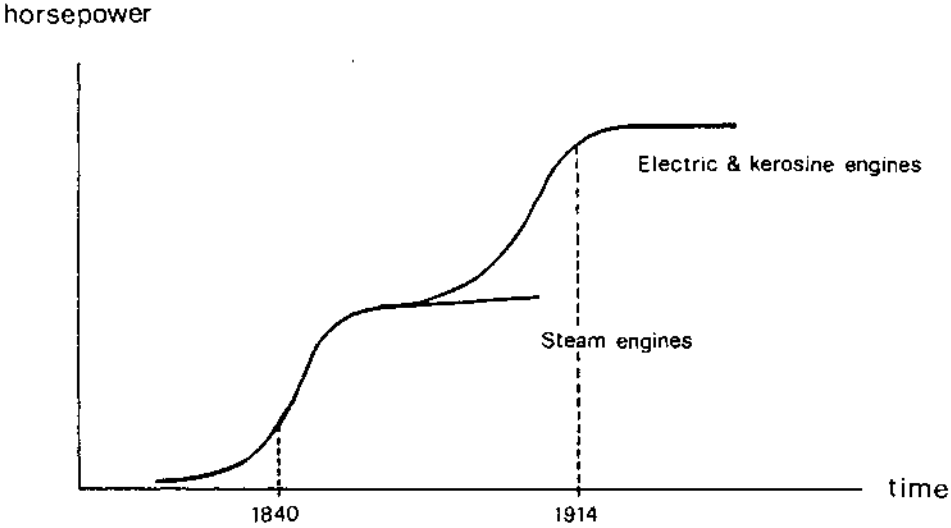
---

[3] A more profound discussion of the statistical estimation of the model is found in Kærgård (1988) where different statistical assumptions are tested, but the main conclusion is the one given here.

could be divided into different phases. Hyldtoft's hypothesis, as mentioned in the introduction to this paper, is that there are long-run fluctuations in the economy, the Kondratieff-waves. In this section a theory of diffusion of steam engines and electric and kerosene engines will be in focus. The theory will be the topic of statistical testing and its relation to the Kondratieff-waves will be discussed.

In the theory of diffusion it is normally assumed that the development is described by a s-shaped curve: At first the development is accelerating, but later it is characterized by a slower approximation to a saturation point. This picture is consistent with what is known from Hyldtoft and others, namely that the high growth rate in the first phase (1840-65) is caused by the production of steam engines, and the high growth rate in the third phase (1896-1914) is derived from electric and kerosene engines. Is so the development should be similar to the picture in fig. 5.

Fig. 5



As seen in the earlier section of this paper there are very few data from the latter part of the period, and statistical analysis should consequently be concentrated on the period 1840-75, as in section 2.

For the sake of simplicity a logistic diffusion function is chosen. The function:

$$HP_i = \frac{K}{1 + be^{-aKi}} \tag{5}$$

where HP_i is mechanical power in year i, K the saturation level, and a and b are parameters, is rather inflexible (it is symmetric around the point where $HP_i = K/2$), but it is simple to work with, because it could be linearized (and then visually tested) as:

$$\Delta HP_i/HP_i = [aK] - a\,HP_i \tag{6}$$

The growth rate ($\Delta HP_i/HP_i$) is a linear function of the stock HP_i. For the whole period 1840-1875 the function does not look attractive. Until 1855 there are unsystematic fluctuations, and after 1868 the growth rates are too high, perhaps as a consequence of the introduction of the new type of engine. A logistic function of the form (6) is hence estimated for the shorter period 1855-1868. The result is:

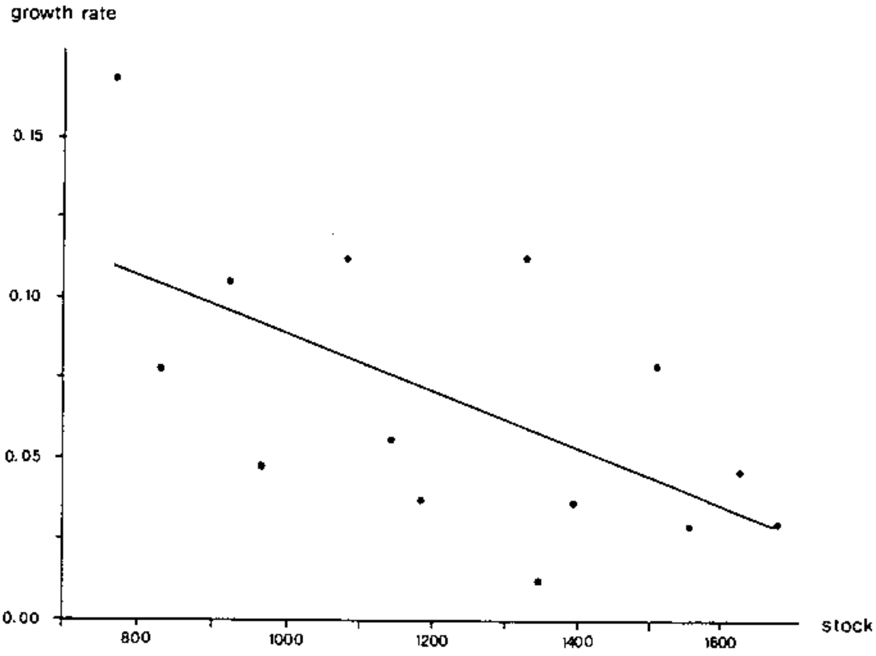
$$\begin{aligned} \Delta HP_i/HP_i = {} & 0.177 - 0.000088 HP_i \\ & (0.042)\ (0.000033) \end{aligned} \tag{7}$$

$$R^2 = 0.37 \qquad D.W. = 2.81 \qquad N = 14$$

The relationships and the observations are shown in fig. 6.

It is seen that the degree of explanation is rather low (0.37), but this is to some extent a consequence of a negative autocorrelation as indicated by the Durbin Watson test, and this is a simple consequence of the use of a variable like ΔHP_i. If big investments in engineering are made in a certain year, common sense will tell us that most of the needs for engine power in the firms are fulfilled, so that the investment in the next period will

Fig. 6
THE DEVELOPMENT IN MECHANICAL
POWER 1855-1868



be small. These are arguments in favour of some sort of smoothing. If one uses the average of a period's growth rate and the next period's growth rate as dependent variables, R^2 will change to 0.66 and if one uses period i and i−1 instead of i+1, then R^2 will be 0.73.

If one tests the residuals for indications of mis-specifications, none of these are found. Mean and variance in the first half of the sample period are not significantly different from those in the latter half. Nor is there any difference between mean and variance of the residuals in the middle part of the sample compared with residuals from the first and the last years.[4] It seems

---

[4] The t-values for testing a difference between the means in two samples is never bigger than 0.7, and the F-value for testing a difference in variance is never bigger than 1.75. There is absolutely no indication of significant differences.

as if the estimated model is valid, but is characterized by a lot of unsystematic noise.

To sum up, one could say that the development straddling the characteristic breakpoint around 1860 may be explained by a theory of a logistic diffusion process, and that it is possible to estimate such a process, but that the estimation results are dimmed by a lot of unsystematic noise. A diffusion process for steam engines seems to be the most convincing theory in order to explain the development in mechanical power for the period 1840-75. Compared to a model founded on Kondratieff's and Schumpeter's theories (as advocated by Hyldtoft, 1984) the difference is perhaps minor, but it is not yet documented that the development in mechanical power has parallels in the general process of development or at least in the development in the stock of capital. In the next section this is investigated by means of some other indicators.

4. Alternative development indicators

In this section several potential indicators of the development in the Danish capital stock for the period 1840-1914 are taken into consideration. As no obvious possibilities are available a broader set of possibilities is considered, and the attempt is made to let the statistical analysis itself to determine the most suitable.

The possible indicators are the ones for mechanical power (see fig. 1) MHP, two series for the amount of fire insurance (the difference between them is due to the included area, see Cohn, 1958, and Falbe-Hansen & Scharling, 1885) FIRE 1 and FIRE 2, and a series for the real value of loans from the private banks in Copenhagen (see Danmarks Statistik, 1969) LOAN.

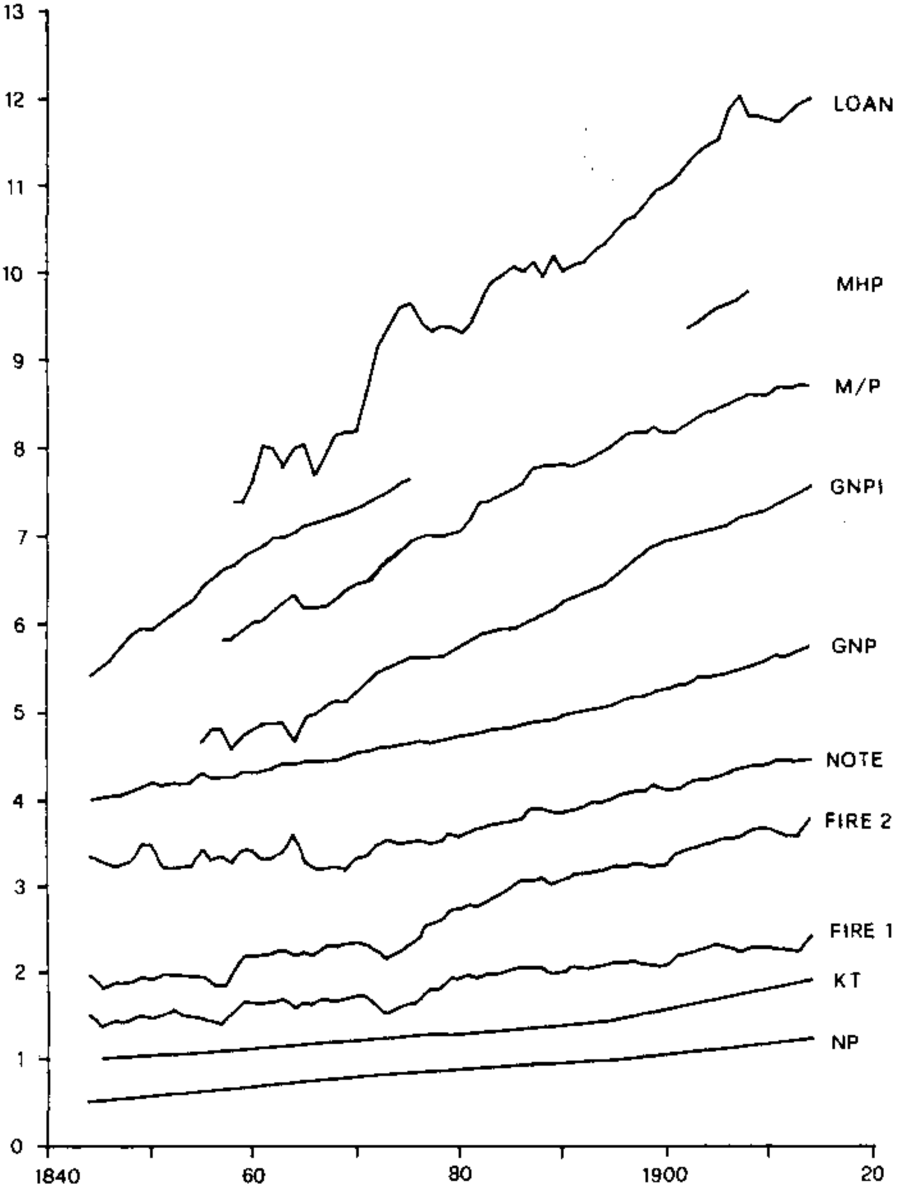
Some series of national accounts data for Denmark are also used. These are gross national product GNP and value added in manufacturing industries GNPI, both in real terms and both taken from Hansen, 1974. A series for the capital stock is calculated from Hansen's investment series, KT.

The quality of the Danish national accounts for this period is debatable, and some data which are more well-defined (but perhaps less relevant for our purpose, too) are taken into consideration. These are the population in the productive age groups (the 15-65 year olds taken from Hansen, 1974), NP, the real value of the note circulation, NOTE, and the real Danish money stock, M/P (both taken from Hansen 1968).

All the series are shown in fig. 7. The idea in the following is to use the factor analysis and the theory of principal components to find some common movements in the series. The method is explained in, for instance, Theil, 1971, and Koutsoyiannis, 1977. The main principle is to construct an index which includes a maximal part of the variation in all the series. This index is found as an eigenvector corresponding to the biggest eigenvalue in the (X'X)-matrix, where X is the data matrix. The second principal component is then an index which takes the maximal part of the remaining variation, and is uncorrelated with the first component. This is the eigenvector corresponding to the second biggest eigenvalue. One can continue until there are as many principal components as series in X and then all the variation in X is determined by the principal components. But normally only a few principal components are necessary for determining the main part of the variation in x. In Theil's analysis of 17 series for the American economy for the period 1922-38 (see Theil, 1971, pp. 50-55) he found that only 3 components are necessary for a proper explanation of the series. Furthermore each of his three components has a straight — forward interpretation — one is an expression for the general income level, one for the growth in income, and the last is a linear trend.

This technique needs a complete X-matrix, and as it is seen in fig. 7 this is not possible for the whole period and with all the series included. It is chosen to use a selection of series around the interesting breakpoint about 1860, and an X-matrix consist-

Fig. 7
INDICATORS FOR THE DEVELOPMENT



Source: All variables are logarithmic transformed. A scale correction is added to the series which means that it is impossible to read the level from the figur. The notation is explained in the text.

ing of the series KT, MHP, FIRE1, FIRE2, GNP, NOTE and NP for the period 1846-1875. The principal components are calculated and the correlation between them and the series in X is shown in table 1. It is seen that all the series — except the note circulation, NOTE — are strongly correlated with the first component. The second component is correlated with the note circulation, but with no other variables. There is no remaining variation for the last 5 components.

The same conclusion could be found in table 2 where eigenvalues corresponding to the components are shown. These measures show how much each component explains; the eigenvalue is the sum of correlation coefficients for each component in table 1 squared. The part of the variation in X which is explained by the first n components is shown, too. It is often mentioned as a rule-of-thumb that the eigenvalue should exceed one for the necessary components.[5] Table 2 confirms that only 2 components are necessary for a satisfactory explanation.

Only the first component is then possible as an estimate of a general development index. This component is shown in fig. 8 (the variable is normalized to mean zero and deviation one). The index is rather trended but with a few years of extreme growth just before 1860. There is a break in the trend, but in contrast to fig. 1 there is no change in the growth rate in the first part of the curve, compared to the latter part. There is a change in the level in 1858/1859, and the development is explained by a dummy for such a change, and by a trend:

$$\begin{aligned} PC1_i = {} & -157.0 + 0.084\,i + 0.56\,d \\ & \quad (9.4) \quad (0.005) \quad (0.09) \\ & R^2 = 0.99 \qquad i = 1846\text{-}75 \qquad D.W. = 1.51 \end{aligned} \tag{8}$$

where $PC1_i$ is the development index from fig. 8 for the year i and d is a dummy variable which is 0 for 1846-58 and 1 for

---

[5] See Koutsoyiannis, 1977 pp. 433-434 for a discussion of the possibility for testing.

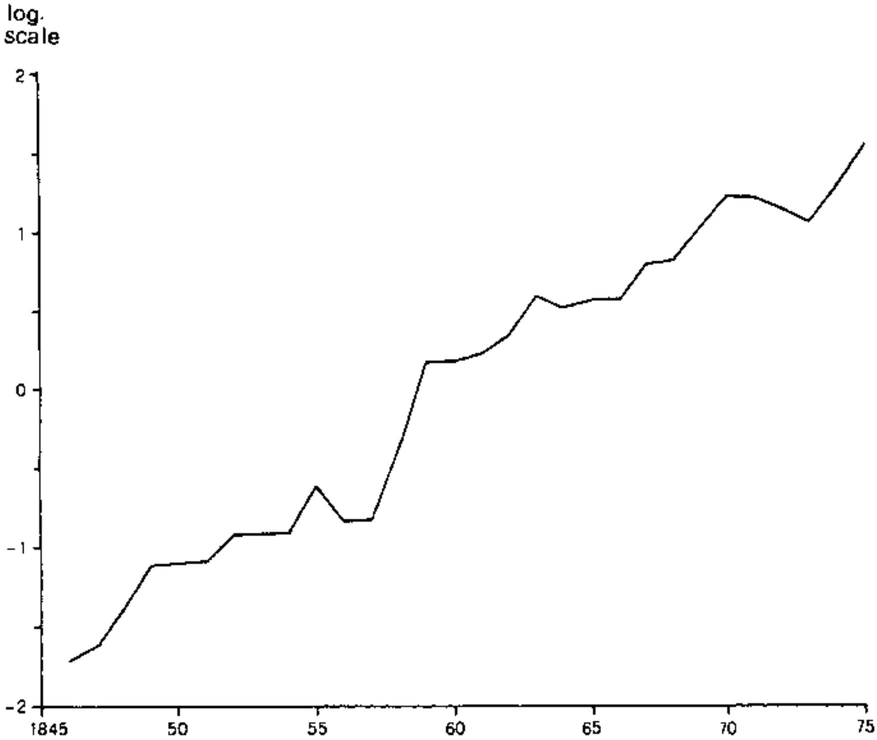
Table 1
CORRELATION BETWEEN THE PRINCIPAL COMPONENTS AND THE VARIABLES
(Factor Loadings)

Principal component no. Variable, logarithmic transformed	1	2	3	4	5	6	7
Capital stock, KT	0,98	0,05	0,18	0,01	0,02	0,03	0,02
Mechanical power, MHP	0,98	0,06	0,15	−0,08	0,06	−0,04	0,00
Fire insurance amount, Copenhagen center, FIRE1	0,82	−0,43	−0,36	−0,08	−0,05	0,01	0,00
Fire insurance amount, Copenhagen FIRE2	0,93	−0,22	−0,26	0,10	−0,06	−0,01	−0,00
Gross National Product, GNP	0,97	0,14	0,14	0,05	−0,11	−0,02	0,00
Real note circulation, NOTE	0,33	0,90	−0,28	−0,01	0,00	0,00	−0,00
Population 15-65 year, NP	0,98	0,03	0,17	−0,01	0,01	0,03	−0,02

Table 2
EIGENVALUE AND EXPLAINED PART OF THE VARIATION

Principal component no. J	Related eigenvalue	Part of variation explained by the first j principal components
1	5,498	0,785
2	1,069	0,938
3	0,382	0,993
4	0,025	0,996
5	0,021	0,999
6	0,004	1,000
7	0,001	1,000

Fig. 8
DEVELOPMENT INDEX



Source: The index is the first principal component of the series on fig. 7.

1859-75. This relation is better than all possible combinations of separate trends of the type in fig. 3, as evaluated with a F-test statistics.

All the series in X are rather trended and a differentiation of the series could then be a natural procedure. If the analysis from table 1 and table 2 is repeated for the matrix ΔX the results will be as shown in table 3 and table 4. These results are quite different from the former. The first 4 components have eigenvalues bigger than one, and all 7 components should be used before the explained part of the variation exceeds 0.99: compared to the case in table 2 where this was reached with only 3 components.

Table 3
CORRELATION BETWEEN THE PRINCIPAL COMPONENTS AND THE VARIABLES
(Factor Loadings)

Principal component no. Variable, (growth rate = dif. ln X)	1	2	3	4	5	6	7
Capital stock, KT	0,37	0,40	0,03	0,73	0,38	0,14	0,02
Mechanical power, MHP	−0,10	−0,08	−0,83	0,40	−0,37	0,04	−0,02
Fire insurance amount, Copenhagen center, FIRE1	−0,90	0,34	−0,10	−0,02	0,08	0,03	0,23
Fire insurance amount, Copenhagen, FIRE2	−0,91	0,27	0,02	−0,02	0,11	0,19	−0,21
Gross National Product, GNP	−0,45	−0,70	−0,08	0,28	0,29	−0,07	−0,03
Real note circulation, NOTE	−0,02	−0,90	−0,04	−0,05	0,12	0,41	0,05
Population 15-65 year, NP	0,30	0,20	−0,68	−0,48	0,42	−0,02	−0,02

Table 4
EIGENVALUE AND EXPLAINED PART OF THE VARIATION

Principal component no. J	Related eigenvalue	Part of variation explained by the first j principal components
1	2,090	0,299
2	1,693	0,540
3	1,171	0,708
4	1,008	0,852
5	0,572	0,933
6	0,363	0,985
7	0,104	1,000

This is an indication of the fact shown in table 5 that the correlation matrix for the ΔX-matrix generally consists of very low figures.

Table 5
CORRELATION MATRIX FOR THE VARIABLES

	Capital	Power	Fire insu., Center	Fire insu., Cph.	GNP	Notes	Population
KT	1.00	0.6	−.18	−.18	−.19	−.30	−.03
MHP		1.00	.11	.02	.16	.05	.17
FIRE1			1.00	.88	.17	−.25	−.09
FIRE2				1.00	.18	−.14	−.18
GNP					1.00	.51	−.23
NOTE						1.00	−.09
NP							1.00

The conclusion must be that with a broader set of development indicators, an index could be constructed for a common fluctuation (except for the note circulation), but this index has other characteristics than the series for mechanical power. The common trend is, however, only a very rough picture of the development, and if one is using growth rates instead of levels the common fluctuation disappears.

5. Conclusion

The paper discusses the economic growth in Denmark for the period 1840-1914, with special reference to the Copenhagen area. It is shown that if mechanical power is used as a growth indicator the development could be divided into different phases, as three different growth functions (for the three periods 1840-1865, 1865-1896 and 1896-1914) describe the development significantly better than one common growth function. The statistically optimal choice of demarcation point between the two first periods is found to be 1858, which is remarkably earlier than the 1865 used by historians.

The change from high to low growth rate is explained by a logistic diffusion function for steam engines, estimated for the period 1855-1868.

But this is based only on data for mechanical power, and if other indicators such as Danish national income figures, the amount of fire insurance and note circulation are taken into consideration the conclusion changes dramatically. If the series are combined by the methods of principal components no indication of a change in growth rate is found. There are indications of a rather complex picture with different movements in the growth rates for the different series. It seems as if one should be very careful with conclusions based on only a few indicators (for instance mechanical power). The long waves in the mechanical power data seem to be a special event and not a sufficient indication for a general Kondratieff-cycle in the Danish economy.

REFERENCES

Brown R.L., J. Durbin and J.M. Evans (1975). 'Techniques for Testing the Constancy of Regression Relationships over Time', *Journal of the Royal Statistical Society*, series B, pp. 149-192.

Cohn E. (1958). *Privatbanken i KjØbenhavn gennem Hundrede Aar 1857-1957*, II. Half volume, Copenhagen.

Danmarks Statistik (1969). Kreditmarkedsstatistik, *Statistiske undersØgelser nr. 24*.

Falbe-Hansen V. and W. Scharling (1885). *Danmarks Statistik I*, Copenhagen.

Farley J.V., M. Hinich and T.W. McGuire (1975). 'Some Comparisons of Tests for a Shift in the Slopes of a Multivariate Linear Time Series Model', *Journal of Econometrics*, pp. 297-318.

Goldfeld S.M. and R.E. Quandt (1976). 'Techniques for Estimating Switching Regression', in Goldfeld and Quandt (ed.): *Studies in Nonlinear Estimation*, Cambridge, Massachusetts.

Hansen S.Aa. (1968). Perioden 1818-1914, in K.E. Svendsen, S.Aa. Hansen, E. Olsen and E. Hoffmeyer, *Dansk Pengehistorie*, The Bank of Denmark, Copenhagen.

Hansen S.Aa. (1970). *Early Industrialization in Denmark*, Copenhagen.

Hansen S.Aa. (1972-74). *Økonomisk vækst i Danmark I-II*, Copenhagen.

Hyldtoft Ole (1984). *KØbenhavns Industrialisering 1840-1914*, (with English summary), Herning.

Johnston J. (1984). *Econometric Methods* (3. edition), New York.

Koutsoyiannis A. (1977). *Theory of Econometrics* (2. edition), London.

Kærgård N. & T.V. Rasmussen (1981). 'A growth model of the Danish Economy', in: Janssen, Pau and Straszak (ed.), *Dynamics Modelling and Control of National Economics*, Oxford.

Kærgård N. (1988). Kinked Exponential Model with Unknown Breakpoints, (unpublished), Copenhagen.

Quandt R.E. (1960). 'Test of the Hypothesis that a linear regression system obeys two separate regimes', *Journal of The American Statistical Association*, pp. 324-330.

Theil H. (1971). *Principles of Econometrics*, Amsterdam.

Willerslev Rich (1952). *Studier i dansk industrihistorie 1850-1880*, (with English summary), Copenhagen.

Willerslev Rich (1954). 'Træk af den industrielle udvikling 1850-1914', *NationalØkonomisk Tidsskrift*, p. 242-262.