

# Contribution to a Model of Very-Long-Term Economic Development

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## ABSTRACT

This paper presents a model of very-long-term economic development, distinguishing between an epoch of exogenous economic development, corresponding to traditional economies, in which structural transformations were rare and exceptional, and an epoch of endogenous economic development, corresponding to Kuznets' "modern economic growth", in which structural transformations have become frequent. Within the framework presented here, it is possible to explain modern economic growth as a consequence of the fact that scientific and technological research is a superior good whose results promote structural transformations in the economy that enlarge its viability zone. Thus, the very fact that per capita income exceeded some threshold levels triggered endogenous economic development on a global scale. Episodes of divergence and convergence among national and regional economic systems within the contemporary world economy can be at least partially explained by feedback effects of the economic development processes.

## Introduction

Modern economic growth (Kuznets, 1966) has been a challenge for economics since the discipline's inception. Smith (1776) already identified two engines of economic growth: division of labour and capital accumulation. Ricardo (1817) added comparative advantage. Schumpeter (1911) completed the standard set of dynamic elements by introducing the concept of innovation, using it to explain simultaneously long-term growth and short-term fluctuations of the capitalist economy.

The best-known neoclassical model of economic growth (Solow, 1956) combined these four engines of economic and population growth to explain productivity growth mainly as resulting from increases in the capital-labour ratio and from technological change. In this context, differences in growth rates across national economies within the contemporary world economy could be explained by differential (exogenous) rates of saving and technological change.

A number of developments flowed from the Solow model. Solow's Nobel lecture (Solow, 1987) is an interesting survey of these developments, which produced what can be considered the standard neoclassical growth theory currently presented in economics textbooks.

Other recent developments in growth theory grappled with the great diversity of national economic growth rates within the contemporary world economy, which seemed impossible to explain exclusively within the framework of a Solow-type model. Endogenous growth theory stressed a number of (endogenous) factors that might account for different rates of saving and technological change: human capital accumulation through formal education or learning-by-doing (Lucas, 1988); increasing returns and openness of the economy (Rommer, 1990); and government policy (Rebelo, 1991). Neo-institutionalist growth theory added institutional settings to this list (Hodgson, 1994). Acemoglu and Robinson (2012) is a recent attempt to present an applied synthesis of this approach. Nor should we fail to mention a neo-Schumpeterian perspective on economic growth, which has sought to analyse the waves of innovation during the epoch of the capitalist world economy (for an overview, see Freeman and Louçã, 2000).

However, all of these theoretical approaches may appear defective when a longer time span is considered. Angus Maddison summarised the main facts that emerge from his millennial perspective (Maddison, 2001) as follows: "From the year 1000 to 1820 the advance in per capita income was a slow crawl – the world average rose about 50 per cent. Most of the growth went to accommodate a fourfold increase in population.

“Since 1820, world development has been much more dynamic. Per capita income rose more than eightfold, population more than fivefold” (p. 27).

This raises the need for a re-examination of growth theory in order to account for the possibility of very-long-term stagnation being followed by a gradual acceleration and then a sudden burst of rapid though uneven growth. The task was attempted in Galor and Weil (2000) and Lagerlof (2006), which presented a unified growth theory stressing the role of population and technology (developments in this analytical approach were summarised in Galor, 2008).

This paper seeks to contribute to this effort. Sections 1 to 5 present a model that combines the typical neoclassical and neo-Schumpeterian features in a very-long-run framework. Section 6 lays out the main conclusions and discusses the historical relevance of the model presented.

## 1. A model of a global economy

Let us define a global economy as a set of economic units that are mutually dependent and, as a whole, self-sufficient in their economic activity.

The following paragraphs will present a formalisation of a model of such economies.

### *Production*

Let us suppose that the production possibility set of a global economy during a conventionally defined period (traditionally one year) depends on the following factors:

- a) technology, i.e. the knowledge about productive processes that is effectively put to use in economic activity;
- b) institutions, i.e. the habits and tastes of human beings, and the organisation of economic activity;
- c) the stocks of productive resources.

Let us also suppose that there are three kinds of productive resources:

- a) natural resources, which are not products of economic activity and whose quantities are supposed not to change as a result of economic activity;
- b) produced resources, which are products of economic activity and which are supposed to be destroyed during the conventionally defined period as a result of productive consumption;
- c) human resources, which are not products of economic activity, but whose quantities are dependent on the consumption of goods for survival and reproduction (and can thus be treated together with produced resources for the sake of convenience).

Let us represent: the qualitative variable technology by  $T$ ; the qualitative variable institutions by  $I$ ; each bundle of natural resources by a vector  $g$  with  $n$  components,  $n$  being the number of different natural resources; each bundle of produced and human resources by a vector  $f$  with  $m$  components,  $m$  being the number of different produced and human resources; each bundle of final goods by a vector  $y$  with  $q$  components,  $q$  being the number of different final goods. All quantitative variables are measured in physical units, and  $g$ ,  $f$ , and  $y$  belong to the non-negative orthants of  $R^n$ ,  $R^m$ , and  $R^q$ , respectively.

To each combination  $(T, I, g, f)$  there corresponds a production possibility set formed by the bundles of final goods that may be produced during the conventionally defined period using that combination. This production possibility set will be represented by  $P(T, I, g, f)$ . The relation between combinations  $(T, I, g, f)$  and their production possibility sets is the production function. Its domain is the set of combinations  $(T, I, g, f)$ ; its range is the power set of the set of bundles of final goods.

To each production possibility set  $P(T, I, g, f)$  there corresponds a production possibility frontier  $F(T, I, g, f)$  formed by the elements of  $P(T, I, g, f)$  that are Pareto-efficient within  $P(T, I, g, f)$ .

The production possibility set that corresponds to the combination  $(T, I, g, f)$  available to an economy at the beginning of period  $t$

includes many potential bundles of production. Of course, only one of them will be the actual or effective production during the following conventionally defined period. Whenever necessary for the sake of clarity, the situation of the variables  $T$ ,  $I$ ,  $g$ , and  $f$  at the beginning of the period  $t$  will be denoted by  $T_t$ ,  $I_t$ ,  $g_t$ , and  $f_t$  and the actual or effective production during period  $t$  will be denoted by  $y_t$ .

Let us postulate that all production possibility sets  $P(T, I, g, f)$  have the following properties:

[p1]  $P(T, I, g, f)$  is non-empty, because it includes at least the origin.

This postulate means it is always possible not to produce. Of course, such a situation is not an interesting one from an economic point of view.

[p2]  $P(T, I, g, f)$  is closed.

[p3]  $P(T, I, g, f)$  is convex.

[p4]  $P(T, I, g, f)$  is bounded.

[p5] Natural, produced and human resources are indispensable.

If all the components of  $g$  or  $f$  are zero,  $P(T, I, g, f)$  coincides with the set whose only element is the origin, i.e. no production is possible.

This postulate implies that:

- i) the economy deals with produced final goods, not with gifts of nature;
- ii) the economy does not work without human productive activity;
- iii) the economy is not a pure result of human activity.

Let us also postulate that  $P(T, I, g, f)$  varies with  $f$  according to the following properties.

[p6] Continuity.

If  $f$  is the limit of a sequence of  $f_i$ ,  $P(T, I, g, f)$  is the limit of the sequence of  $P(T, I, g, f_i)$ .

[p7] Increase.

If no component of  $f_0$  is greater than the homologous component

of  $f_*$ ,  $P(T, I, g, f_0)$  is a sub-set of  $P(T, I, g, f_*)$ . If all components of  $f_0$  are smaller than the homologous components of  $f_*$ ,  $P(T, I, g, f_0)$  is a proper sub-set of  $P(T, I, g, f_*)$ .

[p8] Concavity.

If  $f$  is a convex linear combination of  $r$  vectors  $f_i$  with coefficients  $a_i$ , the set of vectors that can be obtained by convex linear combinations of  $r$  vectors  $y_i$  such that  $y_i$  belongs to  $P(T, I, g, f_i)$  is a sub-set of  $P(T, I, g, f)$ .

[p9] Limitation by a homogeneous function of degree  $p$ , with  $0 < p < 1$ .

Let  $kf$  be the vector whose components are those of  $f$  multiplied by a constant  $k$ .  $P(kf)$  is a sub-set of the set of vectors that can be obtained by multiplying the elements of  $P(f)$  by  $k^p$ .

This limitation can be understood as a consequence of the decreasing marginal returns of produced and human resources, given the constancy of natural resources.

### *Investment*

Let us define gross investment as the production of produced and human resources during the conventionally defined period. Gross investment consists of two parts: depreciation or necessary consumption, i.e. the part needed to restore the previous stocks of produced and human resources; and net investment, corresponding to a change in the previous stocks of produced and human resources. Of course, the components of the depreciation vector must be positive or null, according to the homologous components of the vector of previous stocks of produced and human resources, whereas the components of the net investment vector may be positive, null, or negative.

To each combination  $(T, I, g, f)$ , there corresponds a necessary consumption vector, which is the bundle of final goods that must be consumed to restore the previous stocks of produced and human re-

sources. This necessary consumption vector will be represented by  $C(T, I, g, f)$ . The relation between combinations  $(T, I, g, f)$  and their necessary consumption vectors is the necessary consumption function. Its domain is the set of combinations  $(T, I, g, f)$ ; its range is the set of bundles of final goods.

Let us postulate that the necessary consumption function is a linear application. This means that if  $T, I$ , and  $g$  are given, there is a matrix  $A(T, I, g)$  with  $q$  lines and  $m$  columns, such that:

$$[p10] C(T, I, g, f) = A(T, I, g) f$$

We shall also postulate that:

[p10a] All elements of  $A$  are non-negative.

[p10b] At least one element of  $A$  is positive.

This means that the replacement of depreciation is not a gift of nature, because it implies the following theorems:

[t1a]  $C(T, I, g, f)$  belongs to the non-negative orthant of  $R^q$ .

[t1b]  $C(T, I, g, f)$  is not the origin unless  $f = 0$ .

### *Surplus*

What is left of final goods after necessary consumption has been subtracted is called surplus.

To each combination  $(T, I, g, f)$ , there corresponds a set of potential surpluses formed by the bundles of final goods that may be obtained as surplus during the conventionally defined period using that combination. This set of potential surpluses will be represented by  $S(T, I, g, f)$ . The relation between combinations  $(T, I, g, f)$  and their sets of potential surpluses is the surplus function. Its domain is the set of combinations  $(T, I, g, f)$ ; its range is the power set of the set of bundles of final goods.

In practice, only one of the potential surpluses of  $S(T, I, g, f)$  is the actual surplus in period  $t$ , depending on which of the potential productions of  $P(T, I, g, f)$  is the actual production. The notation  $s_t$  will be reserved for this actual surplus.

The set of potential surpluses can be obtained by subtracting the vector  $C(T, I, g, f)$  from each element of  $P(T, I, g, f)$ . Of course,  $s_t = y_t - C(T, I, g, f) = y_t - A(T, I, g, f)f$ .

$S(T, I, g, f)$  necessarily includes elements outside the non-negative orthant. It may or may not include elements of the non-negative orthant. Surpluses that belong to the non-negative orthant will be called non-negative surpluses; non-negative surpluses that do not coincide with the origin will be called positive surpluses.

### *Viability*

Combinations  $(T, I, g, f)$  can be divided into three types according to the relation between necessary consumption and the production possibility set:

- a) if the necessary consumption does not belong to the production possibility set, the combination  $(T, I, g, f)$  is not viable, i.e. the economic situation cannot stay in a steady state with the given technology, institutions and stocks of natural, produced and human resources. In fact, the surplus of at least one final good is negative;
- b) if the necessary consumption belongs to the production possibility set and the production possibility frontier, the combination  $(T, I, g, f)$  is viable, i.e. the economic situation can stay in a steady state with the given technology, institutions and stocks of natural, produced and human resources, and it is also blocked, i.e. actual production will be equal to necessary consumption and actual surplus will be the origin in every period of the preservation of the stocks of produced and human resources, while no net investment is possible;
- c) if the necessary consumption belongs to the production possibility set but not to the production possibility frontier, the combination  $(T, I, g, f)$  is also viable, i.e. the economic situation can stay in a steady state with the given technology, institutions and stocks of natural, produced and human resources, but it is unblocked, i.e. actual production and actual surplus may vary and net investment is possible.

Before proceeding further, it is important to make a number of remarks about the operational adequacy of some potentially controversial assumptions of the model.

### *Self-sufficiency*

The assumption of self-sufficiency used to define global economies may be criticised because it restricts the adequacy of the model to only a few uninteresting cases, such as those of very small closed economies or the economy of mankind as a whole.

I believe that this is not the case, however. A self-sufficient economy can be operationally defined as one that has only relatively few and unimportant external economic relations. According to this definition, many interesting cases fall within the scope of the model, as will be shown in section 6.

### *Continuity of the economic process*

The assumption of a conventionally defined period, during which produced resources are consumed in order to produce a bundle of final goods that are used to satisfy human wants and to accumulate a new stock of produced resources, may be criticised because it interrupts the continuity of the economic process.

However, this assumption is a classical one in economic theory, perhaps because of the predominant role of agriculture in traditional economic activity and the annual cycle of agricultural life in temperate zones. It is not a critical problem for the operational adequacy of the model, even for economies in which agriculture has lost its predominant role, as long as the conventionally defined period is not too long.

### *Stocks of productive and human resources and flows of productive services*

The distinction between the stocks of productive and human resources and the flows of productive services provided by these pro-

ductive and human resources is a classical one in economic theory, but it is overlooked in the model presented in this essay.

This reduces the realism of certain standard postulates, such as continuity or concavity of the production function. However, it is useful to simplify the formalisation and to stress the fundamental aspects of long-term evolution.

### *Materials and instruments*

The definition of produced resources presented above does not make a distinction between materials and instruments. However, the stocks of materials are wholly consumed in each production process, while the stocks of instruments are only gradually exhausted.

Thus, the assumption of full consumption is appropriate for materials, but is not realistic for instruments if the conventionally defined period is a short one. To simplify the discussion, instruments existing at the end of a conventionally defined period can be considered as final goods ready to be invested in order to restore the previous stocks. This necessitates some adjustments in order not to overlook ordinary depreciation, but it is quite satisfactory for our purposes.

### *Human resources*

The aim of economic activity is to produce the final goods needed to satisfy the wants of human beings. However, the work of human beings is also a resource for economic activity.

Human resources are treated in this essay in the same way as produced resources of the instrument type. It is impossible to treat them as natural resources, because human beings need to consume sufficient quantities of final goods to stay alive and be able to work in the short run, as well as to maintain their number in the long run. To treat them in the same way as produced resources is not wholly rigorous, because the number of human beings is partly determined

by other demographic factors, which will be considered as institutions in the context of the model presented above. However, a specific treatment would be too complex for the available space. Ad hoc remarks will be made where appropriate.

As a consequence of this decision to treat human resources in the same way as produced resources, gross investment includes a human component. This human component of gross investment comprises the consumption needed to maintain the population at a constant level, which is a part of necessary consumption, and the consumption needed to change the population, which is a part of net investment. The lower bound to the consumption needed to maintain the population at a constant level is set by the biological subsistence minimum, but its socially accepted level is, of course, set by what are called institutions in the context of the model.

#### *Discovery, new uses and depletion of natural resources*

According to the definition given above, natural resources are not products of economic activity, and their quantities are supposed not to change as a result of economic activity. As a consequence of this definition, there is no room for accumulation or decumulation of natural resources.

However, it is undeniable that new stocks of some natural resource may be discovered within the geographical scope of an economy; that new uses may be devised for natural resources; and that overuse of a natural resource may deplete or even exhaust the stock existing at a certain moment. All these possibilities can be accounted for, in the framework of the model presented in this essay, as cases of innovations (specifically geographical innovations), on which more will be said in section 5. Of course, the last case is a kind of negative innovation, which cannot be ruled out, especially if the unsustainability of the trend created by the depletion of the stock of a natural resource is not clearly perceived in the short run.

## 2. A typology of economic changes

Economic changes that may occur in a global economy can be divided into three types:

- a) short-term fluctuations, which are changes in the actual or effective production, without changes in technology, institutions, or the stocks of natural, produced and human resources;
- b) size variations, which are changes in the stocks of produced and human resources, without changes in technology, institutions, or the stocks of natural resources;
- c) structural transformations, which are changes in technology, institutions, or the stocks of natural resources.

Actual economic changes are generally a combination of all these types (or, at least, of short-term fluctuations and size variations). However, the distinction between the three types is important, because their consequences can be very different, as will be shown in sections 3 to 5.

## 3. Short-term fluctuations

According to the definition given in section 2, short-term fluctuations are changes in actual or effective production  $y$ , without changes in technology  $T$ , institutions  $I$ , or the stocks of natural resources,  $g$ , and produced and human resources,  $f$ .

Short-term fluctuations can only occur if  $(T, I, g, f)$  is viable and unblocked. In fact, if the combination  $(T, I, g, f)$  is non-viable, it is impossible to maintain a constant  $f$ , because there are no non-negative potential surpluses; and if the combination  $(T, I, g, f)$  is viable and blocked,  $y$  cannot change without reducing  $f$ , because the only non-negative potential surplus is the origin.

Short-term fluctuations can be of two kinds:

- a) exogenous fluctuations, caused by natural or non-economic social facts, for example weather conditions, epidemic diseases, earthquakes or wars. These facts may be distributed over time in a regular or a random fashion;

- b) endogenous fluctuations, such as those analysed by the theories of business cycles in capitalist economies, caused by the normal interplay of economic mechanisms.

The scope of this paper does not permit any further discussion of the possible origins of short-term fluctuations.

#### *Admissible fluctuations*

Of course, any element of  $P(T, I, g, f)$  may be the actual or effective production, and any element of  $S(T, I, g, f)$  may be the actual or effective surplus. However, if the actual surplus is not non-negative, it is impossible to maintain a constant  $f$ , because the necessary consumption of at least one final good exceeds the actual production of that final good. Accordingly, we must distinguish between admissible fluctuations, which occur with non-negative surpluses, and perturbations, which exceed those limits.

Admissible fluctuations are true short-term fluctuations. Perturbations are not short-term fluctuations as defined above and will be treated in section 4.

#### **4. Size variations**

According to the definition given in section 2, size variations are changes in the stocks of produced and human resources  $f$ , without changes in technology  $T$ , institutions  $I$ , or the stocks of natural resources  $g$ .

These size variations can be of two kinds:

- a) reductions of size, caused by short-term fluctuations of the perturbation type;
- b) increases of size, made possible by the net accumulation of produced and human resources, leading to trend growth.

#### *Short-term perturbations*

If the necessary consumption of any final good exceeds the actual

production of that same final good, the stock of at least one produced or human resource must be reduced. A new combination  $(T, I, g, f)$  will emerge with a smaller stock of at least one produced or human resource, and the production possibility set may or may not be reduced according to [p7]. However, the necessary consumption of at least one final good will be reduced, and, if the old combination  $(T, I, g, f)$  was a viable one, the new combination will also be viable (this is a consequence of [t2a] and [t2c] – see below). As a consequence, the economy may establish a new steady state with a smaller size.

This kind of size variation can occur in any type of economy. Its causes can be any of the causes of endogenous short-term fluctuations, when they take what might be called catastrophic proportions.

### *Trend growth*

If actual surplus is positive, it is possible to allocate the positive quantities of final goods that are not used in necessary consumption in two ways:

- a) unproductive final consumption;
- b) net accumulation of produced or human resources.

Of course, if there is no net accumulation of produced or human resources, we are in the context of short-term fluctuations or short-term perturbations. If net accumulation does occur, a new combination  $(T, I, g, f)$  will emerge with a greater stock of at least one produced or human resource, and the production possibility set may expand or not according to [p7]. However, the necessary consumption of at least one final good will also expand, and it is not possible to ensure that the new combination  $(T, I, g, f)$  is viable. This problem will be fully discussed below.

Increases in the stocks of produced and human resources can be of two types:

- a) balanced increases in the stocks of all kinds of produced and human resources;
- b) unbalanced increases, with some kinds of produced or human resources growing faster than others.

Net accumulation of produced or human resources can only occur if  $(T, I, g, f)$  is viable and unblocked. In fact, if the combination  $(T, I, g, f)$  is non-viable, or is blocked, surplus cannot be positive.

As long as surplus is positive (at least as long as it is clearly positive), it is likely that net accumulation will occur. This can be the consequence of two different facts:

- a) economic growth can increase the average income of the members of a particular society, or the disposable income of a privileged social group. Economic growth is thus favourable to the interests of people who have the possibility to promote it by investing a fraction of the part of their incomes that exceeds the minimum amount required for the normal standard of living in their society and social rank;
- b) conflicts between societies are frequent, and large societies have an advantage over small societies when these conflicts arise. Thus, economic growth decreases the probability of defeat by foreigners and increases the probability of victory over foreigners.

### *Viability zone*

However, trend growth cannot go on forever without bounds. In fact, if technology, institutions, and the stocks of natural resources do not change, there is only an upper-bounded viability zone open to the expansion of a global economy.

Let us define the viability zone of a combination of a technology  $T$ , institutions  $I$ , and stocks of natural resources  $g$ , as the set of bundles of produced and human resources  $f$  that make the combination  $(T, I, g, f)$  viable. This viability zone will be denoted as  $V(T, I, g)$ .

It may be stated as a theorem that if the production function is well-behaved (i.e. it has properties [p1] to [p9]) and the necessary consumption function is a linear application (i.e. it has property [p10]), the viability zone  $V(T, I, g)$  is non-empty, closed, convex, and bounded. The proof of this theorem must be divided into its four natural parts. (To simplify the notation, we will write  $P(f)$  instead

of  $P(T, I, g, f)$ ,  $A$  instead of  $A(T, I, g)$ , and so on, as  $T, I$ , and  $g$  are supposed to be given).

[t2a] The viability zone is non-empty.

The origin belongs to the viability zone, because  $C(0) = A \cdot 0 = 0$  and  $0$  belongs to  $P(0)$  according to [p1].

Of course, such an economy, with no produced resources and no human beings, is of scant interest.

[t2b] The viability zone is closed.

Let  $f$  be the limit of a sequence of vectors  $f_i$  that belong to the viability zone, i.e. such that  $C(f_i)$  belongs to  $P(f_i)$ .

As a consequence of [p10], we have:

$$C(f) = Af = \lim_{f_i \rightarrow f} Af_i = \lim_{f_i \rightarrow f} C(f_i)$$

As a consequence of [p6], we have:

$$\lim_{f_i \rightarrow f} C(f_i) \text{ belongs to } P(f)$$

From these premises we conclude that  $C(f)$  belongs to  $P(f)$ , which means that  $f$  belongs to the viability zone. It is thus proved that the limit of a sequence of vectors that belong to the viability zone also belongs to the viability zone, or that the viability zone is closed.

[t2c] The viability zone is convex.

Let  $f$  be the convex linear combination with coefficients  $a_i$  of  $r$  vectors  $f_i$  that belong to the viability zone, i.e. such that  $C(f_i)$  belongs to  $P(f_i)$ .

As a consequence of [p10], we have:

$$C(f) = Af = A \sum_i a_i f_i = \sum_i a_i Af_i = \sum_i a_i C(f_i)$$

As a consequence of [p8], we have:

$$\sum_i a_i C(f_i) \text{ belong to } P(f)$$

From these premises we conclude that  $C(f)$  belongs to  $P(f)$ , which means that  $f$  belongs to the viability zone. It is thus proved that a convex linear combination of bundles of produced and human resources that belong to the viability zone also belongs to the viability zone, or that the viability zone is convex.

[t2d] The viability zone is bounded.

If the viability zone has only one element – the origin because of [t2a] – the proposition is obvious.

If the viability zone has other elements distinct from the origin, let  $f$  be one of these elements and let  $k$  be a positive real number such that  $k^{1-p} > b$ , where  $p$  is the degree of homogeneity of the upper bound of the production function according to [p9] and  $b$  is the highest ratio between homologous components of any vector of  $P(f)$  and  $C(f)$ . ( $b$  is a non-negative real number because of [p2], [p3], and [p4] and there is always such a  $k$  because  $p$  and  $b$  are given and  $\lim_{k \rightarrow +\infty} k^{1-p} = +\infty$ ).

As a consequence of [p10], we have:

$$C(kf) = A(kf) = k(Af) = k[C(f)]$$

As a consequence of [p9], we have:

$P(kf)$  is a subset of  $L$  where  $L = \{y: y = k^p y_0 \text{ and } y_0 \text{ belongs to } P(f)\}$

From these premises we conclude that  $C(kf)$  does not belong to  $P(kf)$ . In fact, if  $k$  is chosen as explained above, the equation  $k[C(f)] = k^p y_*$  has no roots belonging to  $P(f)$ , because it is equivalent to  $y_* = k^{1-p} C(f)$ , and all the components of  $y_*$  are greater than the corresponding components of  $b \cdot C(f)$ , which implies that  $y_*$  does not belong to  $P(f)$ . It is thus proved that the viability zone has an upper bound in whatever direction we choose to consider.

## 5. Structural transformations

According to the definition in section 2, structural transforma-

tions are changes in technology  $T$ , institutions  $I$ , or the stocks of natural resources  $g$ . Structural transformations can, of course, be divided into three types:

- a) changes in the technology, leading to new production possibility sets with the same institutions and stocks of resources. These changes can be called technological innovations;
- b) changes in the institutional framework of the economy, leading to new production possibility sets with the same technology and stocks of resources. These changes can be called institutional innovations;
- c) changes in the stocks of natural resources, leading to new production possibility sets with the same technology and the same institutions. These changes can be called geographical innovations (this choice of name will be explained below).

The main feature common to all innovations is, of course, that they open up the possibility for breaking the limits to growth, which exist without them, as stated by the theorem proved in section 4.

Actual innovations generally occur in clusters embracing all their types. However, as in the case of economic changes, the distinction between the three types is important, because their consequences can be very different.

### *Technological innovations*

According to the definitions above, technological innovations are changes in the knowledge about productive processes that is effectively put to use in economic activity.

A technology  $T_A$  is said to be superior to a technology  $T_B$  if and only if  $P(T_B, I, g, f)$  is a proper sub-set of  $P(T_A, I, g, f)$  for all institutions  $I$ , and stocks of resources  $g$  and  $f$ . Of course, weaker definitions of the superiority of a technology may be devised by admitting  $P(T_B, I, g, f) = P(T_A, I, g, f)$  for some but not all  $I$ ,  $g$ , and  $f$ , or restricting the scope of these variables under consideration. It is important to stress that a superior technology may include the use of resources or the possibility to produce goods that are unknown to an inferior technology.

It is likely that technological innovations only occur in order to replace an inferior technology with a superior one.

### *Institutional innovations*

According to the definitions above, institutional innovations are changes in the habits and tastes of human beings, or in the organisation of economic activity.

Institutions  $I_A$  are said to be superior from an economic point of view to institutions  $I_B$  if and only if  $P(T, I_B, g, f)$  is a proper sub-set of  $P(T, I_A, g, f)$  for all technologies  $T$ , and stocks of resources  $g$  and  $f$ . Of course, weaker definitions of the superiority of institutions may be devised by admitting  $P(T, I_B, g, f) = P(T, I_A, g, f)$  for some but not all  $T$ ,  $g$ , and  $f$ , or restricting the scope of these variables under consideration.

It is not possible to suggest that institutional innovations only occur in order to replace inferior institutions with economically superior ones, because institutions, unlike technologies, are clearly subject to human preferences for non-economic motives. This is why the economic nature of the definition of superiority was stressed in the case of institutions.

### *Geographical innovations*

According to the definitions above, geographical innovations are changes in the stocks of natural resources.

Bearing in mind the qualifications presented at the end of section 1 concerning discovery, new uses, and depletion of natural resources, the most important variations in the quantities of natural resources derive from the integration of previously separated global economies or from the splitting up of previously integrated global economies. This is the reason for the name given to innovations relating to natural resources.

It can be supposed that the integration of previously separated global economies implies the expansion of production possibility

sets because of the opportunities opened up by the utilisation of comparative advantages. It can also be supposed that the splitting up of previously integrated global economies implies the contraction of production possibility sets because of the opportunities closed by the impossibility of using comparative advantages.

A formal definition of the superiority of a bundle of natural resources is easy to obtain from a suitable adaptation of postulate [p7] to the case of natural resources. Of course, the integration of previously separated global economies (like the discovery of previously unknown stocks of some natural resources, or of new uses for a given natural resource) creates a superior bundle of natural resources; and the splitting up of previously integrated global economies (like the depletion of the existing stock of some natural resource) creates an inferior bundle of natural resources.

## 6. Conclusions

The model presented in this paper is designed to capture the main elements of very-long-term economic growth. So-called traditional economies (Rostow, 1960) were characterised by the exceptional nature of structural transformations. Thus, economic changes were usually restricted to short-term fluctuations and size variations, especially trend growth until the limits of the viability zone were reached. Such traditional economies were the background to the situation that Maddison (2001) described for the first millennium AD, “when world population grew by only a sixth, and there was no advance in per capita income” (p. 17).

The same work by Maddison explains the economic performance of the second millennium AD by “[c]onquest or settlement of relatively empty areas”, “international trade and capital movements” and “technological and institutional innovation” (p. 18). This corresponds to the structural transformations defined above (section 5). “From the year 1000 to 1820 the advance in per capita income was a slow crawl – the world average rose about 50 per cent. Most of the growth went

to accommodate a fourfold increase in population” as a result of the fact that such structural transformations were still rare. “Since 1820, [... p]er capita income rose more than eightfold, population more than fivefold,” thanks to “modern economic growth” based on “the extended application of science to problems of economic production” – the “epochal innovation,” according to Kuznets (1966).

Within the framework presented here, it is possible to explain modern economic growth in this sense as the consequence of two facts:

- a) scientific and technological research is a superior good. This means that scientific and technological research increases quantitatively as income rises, so that economies with higher average per capita income, or with higher individual incomes of some members of the society, are likely to devote more resources to such activity.
- b) the results of scientific and technological research promote structural transformations in the economy. As a consequence, increased resources devoted to scientific and technological research are the basis for technological, institutional and geographical innovations, which enlarge the viability zone of the economy, breaking former limits to growth and allowing further accumulation of produced and human resources.

Given these assumptions, it is possible to conclude that the very fact that per capita income exceeded some threshold levels triggered endogenous economic development on a global scale, by means of increased allocation of resources to scientific and technological research. At the same time, episodes of divergence and convergence among national and regional economies within the contemporary world economy can be at least partially explained by feedback effects of economic development processes.

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