
Schools and Teachers of Commercial Arithmetic in Renaissance Florence

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From the fourteenth to the sixteenth centuries the education of the son of a Florentine patrician was directed toward his professional goals — business in one form or another.¹ Primary education — reading, writing, the study of grammar and the rudiments of Latin — was begun between the ages of five and seven. Already at the age of eight, for example, the son of the diarist Morelli knew how to write and was sending letters to his mother when she was away in the country, and by nine he knew Latin and could read commercial letters.² When they were ten or eleven boys went off to a *scuola d'abbaco*, where they concluded their formal education with a course in commercial arithmetic. Then, at the age of thirteen or fourteen, they were ready for on-the-job training, usually in a local cloth manufactory or banking office. Here they familiarized themselves with the business, learning to handle the cash box and eventually to keep accounts but meanwhile doing « other lowly and dirty tasks » (as we know from the frequently cited observations of the Venetian ambassadors, who were always somewhat surprised that sons of the city's wealthiest patricians should engage in manual labor).³ Some

¹ On mercantile education in Florence see ARMANDO SAPORI, *La cultura del mercante medievale italiano*, in his « Studi di storia economica (secoli XIII-XIV-XV) », I (Florence, 1955), pp. 66-67; and A. FANFANI, *La préparation intellectuelle et professionnelle à l'activité économique en Italie du XV^e au XVI^e siècle*, « Le Moyen Age », LVII (1951), pp. 327-46. A more general discussion is in CHRISTIAN BEC, *Les marchands écrivains à Florence 1375-1434* (Paris, 1967), pp. 383-93; and LAURO MARTINES, *Lawyers and Statecraft in Renaissance Florence* (Princeton, 1968), pp. 78 ff.

² GIOVANNI DI PAGOLO MORELLI, *Ricordi* (Florence, 1956), p. 457.

³ *Relazioni degli ambasciatori veneti al senato*, ed. Arnaldo Segarizzi (3 vols.; Bari, 1912-16), III, Pt. I, pp. 17, 185-86, 210; Pt. II, p. 42. In the late fourteenth century Giovanni Morelli advised that children receive experience working in shops; and in the mid-fifteenth century Cotrugli observed that this was a custom practiced above all in

— probably relatively few, those who had the prospect of entering international banking and commerce — went abroad to work in branch offices. At very early ages, for example, the sons of Alessandra Macinghi negli Strozzi were moving all about Europe from Sicily to the Lowlands, learning about the branch operations of the great commercial establishment of their cousin with whom they were serving their apprenticeships. One of these, Lorenzo, when only sixteen years old, wrote home from Valencia that he had only about three hours of sleep and the rest of the time he spent writing letters for the company.⁴ It must have been a hard life, at least during the years of adolescence and youth; and marriage and settling down to domestic life came relatively late for the Florentine. Meanwhile, there was little opportunity for developing a sense of higher culture. Few boys could have enjoyed the luxury of a tutor to direct their interests to humanistic studies, and even a good knowledge of Latin was probably not widely diffused among the patriciate. « Very few men in that period took up Latin - or were made to take it up », says one fifteenth century writer, « unless they expected to become a doctor, notary, or priest ».⁵ There were of course some learned patricians — and much has been made of them recently — but it is no wonder that Alberti in generalizing about the society as a whole felt compelled to lament the lack of interest in letters among these merchants of Florence.⁶

In this process of growing up a boy's formal secondary education was in most cases limited to the *scuola d'abbaco*. Originally the term *abbaco* referred to a device for calculating by means of disks, beads, counters, etc.; but in Italy after the introduction of the use of arabic numerals and the mathematical work on computations with them by Leonardo Fibonacci in the early thirteenth century, the term *abbaco* came to be used in a general sense for instruments, methods, manuals, schools, teachers or anything else related to the skill of doing computations, especially with reference to practical applications in the mercantile world. In fact, in the title of the first printed book on algorism the « arte dell'abbaco » is equated with the « larte dela merchadantia ».⁷ The simple instrument we think of today when we use the English word abacus with its counters arranged along rods or grooves, was not actually used in Italy in the later middle ages as it was in northern

Florence; see ROBERT S. LOPEZ and IRVING W. RAYMOND, eds. and trans., *Medieval Trade in the Mediterranean World* (New York, 1955), pp. 416 and 422.

⁴ ALESSANDRA MACINGHI negli STROZZI, *Lettere di una gentildonna fiorentina del secolo XV ai figliuoli esuli*, ed. Cesare Guasti (Florence, 1877), p. 30.

⁵ ANTONIO MANETTI, *The Life of Brunelleschi*, ed. Howard Saalman (University Park, Penn., 1970), p. 38.

⁶ *I primi tre libri della famiglia*, ed. F. C. Pellegrini (Florence, 1946), pp. 59-61.

⁷ *Incomincia una practica molto bona et utile a ciaschaduno chi vuole uxare larte dela merchadantia chiamata vulgarmente larte de labbaco* (Treviso, 1478).

Europe,⁸ although, as Saporì has shown, for his actual calculations the Italian merchant of the early Trecento did use a similar method with surfaces of tables and tablets divided into squares and columns on which counters were placed.⁹ It is likely however that even this practice was discontinued by the fifteenth century with the growing sophistication of the merchant's mathematical education.¹⁰

A *scuola d'abbaco*, therefore, was an elementary commercial school where boys learned the basic arithmetic they would need for the business world. In Lucca the commune kept a *maestro d'abbaco* in its employ for purposes of public instruction,¹¹ but everything we know about the schools of Florence indicates they were private establishments. Saporì cites references to boys studying arithmetic presumably at such schools in the 1330's; but as he notes, one of his examples seems to be a kind of apprenticeship with an employee of the Peruzzi bank and not formal instruction in a school.¹² Nevertheless, Villani's famous statistics for 1338 included six *scuole d'abbaco* with a total of between 1000 and 1200 in attendance. The student body of these schools had a broad social range descending from the patriciate down at least as far as small shopkeepers like Luca Landucci.¹³ And they learned their lessons well: anyone who is familiar with the complexity of the arithmetic problems of their monetary systems, international exchange rates and other accounting and business practices and who has tested the accuracy of the calculations in their commercial records can vouch for the effectiveness of the training Florentines received in the *scuola d'abbaco*.

The names of many Florentine teachers of the later fourteenth and fifteenth centuries are known, and from the number of them found abroad one might suspect that Florence was an important center for the training of them.¹⁴ One of the best known is Maestro Giovanni di Bartolo. The son of

⁸ DAVID EUGENE SMITH, *The Influence of the Mathematical Works of the Fifteenth Century upon Those of Later Times*, «The Papers of the Bibliographical Society of America», XXV (1931), p. 149.

⁹ SAPORÌ, *La cultura del mercante*, pp. 72-78, where there is a discussion of how such calculations were made.

¹⁰ FEDERIGO MELIS has emphasized the obsolescence of the abacus in Florence by the fourteenth century and therefore expresses considerable surprise when he reports a request found in the Datini papers for a «tavoletta del gesso, ch'è buona per fare ragioni»; *Aspetti della vita economica medievale (studi nell'Archivio Datini di Prato)*, I (Siena, 1962), pp. 347-48. But although the instrument itself was outmoded, a derivative system of doing calculations with such *tavolette* was still used, as Saporì's evidence clearly shows.

¹¹ PAOLO BARSANTI, *Il pubblico insegnamento in Lucca dal secolo XIV alla fine del secolo XVIII* (Lucca, 1905), pp. 54-57.

¹² SAPORÌ, *La cultura del mercante*, p. 67n. For earlier but unclear evidence, see C. T. DAVIS, *Education in Dante's Florence*, «Speculum», XL (1965), pp. 415-35.

¹³ See note 31 below.

¹⁴ Seven out of the ten recorded *maestri* in Lucca in the fourteenth century were Florentine; BARSANTI, *Il pubblico insegnamento*, p. 239. There are records of a Florentine teaching in Venice at the same time (BARTOLOMEO CECCHETTI, *Libri, scuole, maestri, sus-*

a builder (*muratore*), Giovanni was a student of Maestro Antonio de' Mazzinghi, himself a well-known teacher; and when Mazzinghi died about 1390 Giovanni, who was only nineteen, decided to continue his master's school at Santa Trinita. The other masters in the city were disturbed by what seemed to them the impudence of one still so young at entering their profession, and they conspired to get him to close it. Two of these prevailed on their students — numbering about fifty altogether — to attend the new teacher's courses, telling them that one so young was not in any better position to teach than the students themselves. The teachers' hope, of course, was that these students would expose Giovanni's inability to teach and that he would therefore retire from the profession; but (as one would expect in this kind of anecdote) the students were so impressed with how much they could learn from Giovanni that they abandoned their former teachers for the new school. Giovanni operated this school for many years, one of his most famous students being Paolo Toscanelli. In his declaration for the 1427 Catasto he was still paying seventeen florins rent for a *bottega d'abbaco* at Santa Trinita, although he claimed that as a result of a dislocated hip he had not been able to work for nine years. He died in 1440.¹⁵

Hardly anything is known about the internal organization of a *scuola d'abbaco*. Gino Arrighi has recently published the brief outline of a program for teaching arithmetic found in an early fifteenth century arithmetic treatise written by a Pisan who may have been a *maestro d'abbaco*.¹⁶ It contains some extraordinary detail on the pedagogical procedures by which students could be led to refine their mastery of basic arithmetical operations, and it suggests the kind of didactic program that was most likely followed in such schools. Another document has recently come to light that gives us additional information on how at least one of these schools in Florence was actually organized.¹⁷ It is a contract of employment between a *maestro d'abbaco*

sidi allo studio in Venezia nei secoli XIV e XV, « Archivio veneto », XXXII [1886], p. 355) and another teaching for the commune of Brescia about a century later (GINO ARRIGHI, *Paolo dell'Abbaco e Benedetto da Firenze nei Mss. 946 e 947 « Fondo Italien » della Bibliothèque Nationale di Parigi*, « Bollettino della Unione matematica italiana », ser. 4, II (1969), p. 127.

¹⁵ On Giovanni di Bartolo see GINO ARRIGHI, *Il codice L.IV.21 della Biblioteca degl'Intronati di Siena e la 'bottega dell'abaco a Santa Trinita di Firenze'*, « Physis », VII (1965), pp. 369-400; and Id., *La matematica del Rinascimento in Firenze: l'eredità di Leonardo Pisano e le 'botteghe d'abaco'*, « Cultura e scuola », V (1966), no. 18, pp. 287-94.

¹⁶ Un 'programma' di didattica di matematica nella prima metà del Quattrocento (dal Codice 2186 della Biblioteca Riccardiana di Firenze), « Atti e Memorie dell'Accademia Petrarca di Lettere, Arti e Scienze di Arezzo », nuova serie, XXXVIII (1965-67), pp. 112-28.

¹⁷ Archivio di Stato, Firenze: Notarile antecosimiano, C. 547 (ser Lorenzo Cioli, 1519-1522), ff. 30r-32r.

Yhs

Die 3 decembris 1519 in apotheca, presentibus ser Iohanne Simone de Colle et Andrea Antonii de Bocettis testibus.

Hogni volta Giuliano di Buonaguida della Valle mosterrà a me Francescho di Lio-

and an assistant, drawn up on 30 November 1519 and notarized three days later. The master is Francesco di Leonardo Galigai, who is otherwise known only for a mathematical treatise published just two years later, in 1521. In

nardo Ghaligaio una fine tra lui e Piermaria, la quale di presente à mostra, di mano di detto Piermaria, e che per detta fine si vedrà tra Piermaria e Giuliano esere finito hogni chonvegna e leghame avesino avuto a fare insieme insino a questo giorno della fine, e per detta aparischa Piermaria licenziare detto Giuliano.

E mediante detta licenzia e fine tra Piermaria e Giuliano detto, mascie una chonvegna e achordo tra Francescho e Giuliano sopra iscritti, in questo modo, cioè:

Detto Francescho debbe ricevere detto Giuliano in sua ischuola, e detto Giuliano debbe venire in detta ischuola di Francescho a uso di buono gharzone e oservare e mantenere gli ordini e chostumi di detta ischuola, cioè le mute che per insino al presente giorno ho insegnato a quegli ischolari che l'anno volute imparare, chosì si debbono insegnare a quegli che vogliono per l'avenire inparare in quel modo che agli altri insino a questo dì detto di sopra s'è fatto cioè s'è insegnato.

Anchora in mia ischuola ogni giorno 2 volte si dice certe orazione quando si partono gli scholari, le quale horazione saranno in ischuola iscritte. Detto Giuliano debe mantenere detta usanza sichondo el modo di detta ischuola.

E mediante che detto Giuliano enterrà in detta ischuola di Francescho, si facci uno libro chapacie a ricevere tutti gli scholari di detta ischuola, e chosì tutti iscritti e di che muta è chiaschuno ischolare. E perché detto libro è per l'una parte, chome per l'altra, si paghi a mezzo.

E chosì tutti gli scholari, e di che muta, ciascuno sarà iscritto in detto libro; e perché tutti non sono d'una muta, è necessario parlare muta per muta.

Dicho la prima muta si chiama libretine, nella quale muta vi si chontiene rachorre, multiprichare libretine, e ragione che in atto nessuno non vi si intervenga el partire. E questa muta si chiama libretine, cioè la muta delle libretine si chontiene tutte le ragione che si possono dare, nelle quale chome è detto non vi s'è a intervenire in modo alchuno el partire se none in fare denari soldi, e in soldi lire; e nel modo chonsueto che insino a questo presente giorno s'è oservato d'insegnare, si debbe per lo avenire mantenere d'insegnare. E tutti gli scholari di detta ischuola che detto Giuliano troverà in detta muta, quando l'aranno inparata e che detti enterranno nella prima de' partitori, io Francescho debbo dare a detto Giuliano per ciascuno ischolare, quando enterrà nella prima de' partitori, soldi sette. E se alchuno non venissi di nuovo, detto Giuliano non debbe aspettare che egli abbi inparato detta muta delle libretine avere e' soldi sette detti, ma in chapo di 15 dì della venuta dello ischolare el detto Giuliano debbe avere e' detti soldi sette. E quando detto Giuliano arà auto e' soldi sette della muta detta libretine, non debbe avere altro di detto ischolare se non quando enterrà o sarà nella prima de' partitori, chome da piè si dirà nel capitolo de' partitori.

Doppo la muta detta libretine segue una muta di ragione detta prima de' partitori, nella quale gli scholari fanno regholi e ragione le quali non si partono se none una volta, e tutte le ragione si partono una volta si dice prima de' partitori, bene si multiprichasino più volte, come i' mia ischuola s'è oservato per el passato, si debe hoservare per lo avenire d'insegnare.

E tutti gli scholari che detto Giuliano troverà nella prima de' partitori, bovero fusino u[s]citi o uscissino delle libretine o entrasino in detta prima de' partitori, quando l'aranno inparata di modo che detti entrino nella sichonda de' partitori, allora detto Giuliano debbe avere per ciascuno ischolare soldi cinque; salvo se di nuovo ne venissi nessuno che fussi ito a altra ischuola ho avessi inparato dal padre, allora Giuliano non debbe aspettare avere e' soldi cinque che egli abbi inparato la prima de' partitori, ma si

the dedication of this treatise Galigai states that he had been practicing mathematics for sixteen years: and it would seem from references in our contract to the customs of his school that in 1519 it was already a well estab-

in chapo di 15 dì della venuta di detto iscolare debbe detto Giuliano avere s. cinque per detta prima de' partitori da me Francescho.

E tutti gli scolari che detto Giuliano troverà nella sechonda de' partitori hoverso fussino u[s]citi della prima de' partitori e entrasino o fussino entrati nella sichonda de' partitori, quando l'aranno inparata di modo entrino nella terza de' partitori, allora detto Giuliano debbe avere per ciaschuno scholare s. cinque. E chiamasi la sichonda de' partitori tutte le ragione che si partono dua volte, bene che più volte si multiprichino. E detti scolari in detta muta fanno ogni giorno e regholi, chome è chonsueto, e tutte le ragione che si partono 2 volte, sono della sichonda de' partitori.

E tutti gli scolari che detto Giuliano troverà nella 3^a de' partitori hoverso uscino della sichonda de' partitori e fusino u[s]citi di detta sichonda de' partitori, quando aranno inparato detta 3^a de' partitori di modo entrino ne' rotti, allora detto Giuliano debbe avere per ciaschuno iscolare s. tre, salvo venissimo di nuovo, che sempre in chapo di 15 dì debbe avere la parte gli tocha di quella muta fussi detto scholare. E la terza de' partitori s'intende tutte le ragione si partono tre volte o più, chome in nia ischuola è chonsueto d'insegniare.

E tutti gli scolari che detto Giuliano troverà ne' rotti hoverso uscino della terza de' partitori o fussino u[s]citi quando aranno inparato detti rotti di modo entrino nella reghola delle 3 chose, allora detto Giuliano debbe avere per chiaschuno iscolare s. quatro. E e' rotti s'intendono multiprichare, partire, agugnere, trarre quale più ho quanto piglia, e recharre in parte; non abandonando gli scolari e' regholi.

E tutti gli scolari che detto Giuliano troverà nella reghola delle 3 chose, hoverso uscino o fussino u[s]citi de' rotti, quando aranno inparato detta reghola delle tre chose di modo entrino nelle monete fiorentine allora dello Giuliano debbe avere per ciaschuno iscolare s. quatro. E detta reghola fa dua effetti, cioè detta vende ho sì detta chompera, chome si vede per pasato che s'è insegnato.

E tutti gli scolari che detto Giuliano troverà nelle monete fiorentine hoverso uscino della reghola delle tre chose ho fussino u[s]citi di detta reghola, allora quando aranno inparato le monete fiorentine, detto Giuliano debbe avere per ciaschuno iscolare s. quatro. E chiamasi le monete fiorentine tutto quello che in oro e in arietno, istanpato e none istanpato, s'apartiene, e chosì monete choniate e non choniate.

E chosì tutti gli scolari quando aranno passato le monete fiorentine. detto Giuliano d'essi iscolari non debbe partecipare nulla, ho istati in ischuola ho venuti di nuovo, ma a quegli insegnare tanto sia piacimento di detti scolari.

E chome el guadagno che detto Giuliano trarà di detti iscolari in detti modi, cioè ne' modi che ne' chapitoli di sopra s'è detto, agugnerà a fiorini dodici d'oro in oro inanzi vengha l'anno della venuta di di detto Giuliano in detta schuola, allora Giuliano e Francescho detti fanno nuova chonvegna insino vengha l'anno della venuta di detto Giuliano, cioè chome detto salario che trarà detto Giuliano, chome apari[s]ce ne' soprascritti chapitoli, agugnie, chom'è detto, a f. dodici d'oro in oro, detto Giuliano della muta detta librettine non debbe avere altro che s. tre quando gli scolari l'aranno inparato, cioè s. tre per iscolare di detta muta della librettine, e d'ogni altra delle sopradette mute debe avere apunto s. uno. E questa chonvegna de' tre soldi e un soldo per muta duri infino vengha la venuta de l'anno di detto Giuliano in detta ischuola.

El sichondo anno si seguiti in quel modo del primo infino vengha el guadagno de' f. dodici d'oro in oro. E quando detto guadagno di detti f. dodici saranno venuti nelle mane di detto Giuliano inanzi finisca el sichondo anno, si seghua lo stile del primo anno

lished institution. It was apparently located in the vicinity of Santa Maria Maddalena dei Pazzi, then a convent of Cistercian nuns, for it was here, « in Cestello », that there was an annual religious service for his students. About

di tre soldi nelle libretine e un soldo per ciaschuna altra detta muta, infino vengha el termine del sichondo anno, cioè insino finischa el sichondo anno.

E nel terzo anno s'oservi lo stile e hordine e modo che s'è oservato nel primo anno e nel sichondo anno; e in detti tre anni Giuliano nè Francescho non si possono dividere nè partissi l'uno da l'altro nè muovere altri patti l'uno a l'altro.

E anchora, delle mancie che si trarà di detti ischolari detto Guliano none debbe partecipare alchuna chosa, e sieno di che muta si voglia gli scholari, nè l' nesuno tempo nè di nesuno scholare detto Guliano in detti 3 anni non debe avere alchuna mancia nè altro che ne' soprascritti chapitoli si chontiene, e ogni guadagno si trarà degli ischolari debbe essere di me Francescho, che ò a dare e soprascritti danari a Guliano nel modo che ne' soprascritti chapitoli si chontiene.

E detto Guliano possa tre volte l'anno andare in villa e stare dieci giorni per volta. E io Francescho debbo procurare per lui; e simile, amalando, per uno mese e mezzo gli debo serbare, o da[r]llo a chi per lui fussi, el suo guadagno, e da uno mese e mezzo in là che Giuliano non venissi a schuola, el guadagno si trarà in mia schuola sia tutto di me Francescho infino che Giuliano vi ritorna a insegnare.

E amalando io Francescho, el tempo che a Dio piacesse che io istessi di non potere esercitare la schuola, Guliano per me in quel tempo debe procurare di rasegnarmi quello mi tocha chome quando ero sano. E se in quel tempo che Guliano è chonsuetto d'andare in villa, io Francescho amalassi, detto Guliano debbe avere pazienza insino guarischa, e poi vadi a sua chonsolazione.

Anchora è in mia ischuola usanza che lla matina di Santo Nicholdò, a dì 6 di dicembre, tutti gli scholari veng[h]lino a schuola e andiano a fare chantare la messa del detto santo in Cestello, e ritornano in mia ischuola e dassi panelini benedetti. Detto Guliano questa usanza debbe mantenere.

E se alcuno chontrafarà a detti chapitoli [sic].

E ogni volta che Guliano arà da Piermaria la sopradetta fine, debiano cho' sopradetti chapitoli e chonvegnia andare e chavarne chontrato.

E questi chapitoli sono fatti oggi questo dì 30 di novembre 1519, di mano di me soprascritto Francescho. E se detto Guliano fra uno mese e mezzo chome segue, non arà da detto Piermaria la detta fine, non tenga alchuna chonvegnia ho patto o scritta che si è tra me detto Francescho e l' sopradetto Giuliano, e non si intenda avere a fare nulla l'uno cho' l'altro.

Anchora, quando chavereno instrumento, vi si metta una pena fra l'uno e l'altro di f. venti d'oro in oro.

E quando detto Giuliano si partirà in alchuno modo da detto Francescho, el detto Giuliano non debbe insegnare nella città di Firenze se prima non passa tre mesi della partita di detto Giuliano di schuola di detto Francescho.

Anchora, per oservazione de' soprascritti chapitoli, detto Francesco e Guliano s'obrigono l'uno a l'altro e l'altro a l'uno, e loro redi e beni presenti e futuri, e sotometonsi a qualunque ufiziale o huffizio ho chorte dove ragione si tenessi, chosì in Firenze chome in qualunque altra città, chosì eclesiastica chome secolare, e rinunziano a ogni beneficio che per loro facessi e a ogni istanto ho legge che in suo favore venissi. E per chiarezza di ciò, io Francescho soprascritto mi sono sottoscritto e ò iscritto e' soprascritti chapitoli. E chosì, per la certezza della verità, el soprascritto Guliano si sottoscriverà qui da piè di sua mano, chome è chontento a quanto di sopra è iscritto. Anchora, se alchuno di noy, cioè se Guliano detto e Francescho, chontrafacessino ad alchuna delle soprascritte, cioè

the only other fact we have about Galigai is that he died in 1536.¹⁸ Galigai's new assistant was Giuliano di Buonaguida della Valle, who apparently was still very young (he is called a *garzone*). He previously had had unspecified obligations hereby dissolved with someone identified only as Piermaria.¹⁹

Giuliano was to be employed as a teacher in Galigai's school for three years, and the contract lays out clearly his teaching program. It consisted of seven consecutive courses (*mute*): 1) basic arithmetical operations except division, called the *librettine* (referring to tables used by the student for calculation); 2), 3) and 4) division, divided in three courses according to which the student learned to divide with one, then two and finally three or more digits; 5) fractions; 6) the Rule of Three; and finally 7) the Florentine monetary system. This was the extent of Giuliano's teaching responsibilities. It was undoubtedly the basic curriculum followed by all boys who attended any *scuola d'abbaco*; and it closely resembles (though with less pedagogical detail) the didactic program of the Pisan document published by Arrighi. The program, in short, consisted of basic arithmetic and instruction in the complicated monetary system of Florence. Problems were no doubt cast in the practical terms of a developed trade economy (« e detta reghola [the Rule of Three] fa dua effetti, cioè ho detta vende ho sì detta chompera »); and in the course of working such problems, the student would acquire considerable information about the facts of commercial life — exchange of moneys, loans and interest, partnerships, mensural systems, etc. Information courses as such, however, and courses in the non-mathematical techniques and procedures of basic business practice, such as foreign exchange operations and accounting, were probably not taught.²⁰ These were things a boy

soprascritte chose, rimangha chondannato in f. venti d'oro in oro, standosi a quello che sarà hoservatore di sopra scritte chose.

E io Francesco di Lionardo Ghalighaio, maestro d'abaco, sono chontento a quanto di sopra si chontiene, e per fede del vero mi sono soscritto di mia propria mano oggi questo dì 30 di novembre 1519.

Io Giuliano di Buonaguida della Valle sono chontento a quanto di sopra si chontiene, cioè ne' sopradetti chapitoli e ischritta, e per fede e per fede del vero ho fatti questi versi di mia propria mano hogi, questo dì 30 di novembre 1519.

¹⁸ On Galigai, see notes 33 and 36 below; in the secondary literature the old spelling, Ghaligai, is frequently retained.

¹⁹ Forty years earlier, in 1480, Niccolò Machiavelli studied with a *maestro d'abbaco* identified only as Piero Maria; see below note 22. Could he be the same?

²⁰ Federigo Melis has asserted that basic accounting was also taught in these schools even by the end of the fourteenth century, basing his argument, however, solely on the uniformity of accounting practice in Florence; *Aspetti della vita economica*, p. 344. In an earlier work Melis stated his views much more guardedly, emphasizing the greater importance of apprenticeship over a schooling at least before 1500 (he apparently assumed that the appearance of treatises on accounting meant also its inclusion in school curricula); *Storia della ragioneria, contributo alla conoscenza e interpretazione delle fonti più significative della storia economica* (Bologna, 1950), pp. 608-11. Cf. note 25 below.

learned during his years of apprenticeship immediately following school. Any advanced instruction in mathematics was possibly given by Galigai himself, and if so, it was probably oriented to those select few, above all his own apprentices, whose interest was more purely mathematical than commercial.

Giuliano was paid on the basis of a rate per student for each of the seven courses; the rates varied for each course, and the entire schedule of rates changed after his total earnings reached twelve florins for any one year of service. The rates were as follows (in *soldi di piccioli*):²¹

Course	First rate	Second rate
<i>Librettine</i>	7 s.	3 s.
Division 1	5 s.	1 s.
Division 2	5 s.	1 s.
Division 3	3 s.	1 s.
Fractions	4 s.	1 s.
Rule of Three	4 s.	1 s.
Monetary system	4 s.	1 s.
	Total	Total
	32 s.	9 s.

Presumably the rates reflect the relative time involved in the teaching of each course. It is impossible to estimate Giuliano's probable annual income inasmuch as we have no idea of the length of the courses, the number of students, and their turnover. The twelve florins cut-off figure must have been something like a guaranteed minimum salary; it would in fact have been equivalent to the annual income of an unskilled worker who was able to work fulltime, that is, close to an absolute minimum wage in Florence. Giuliano received his compensation entirely in cash from Galigai, and he was not permitted to accept any money from the students. The contract was drawn up strictly for his employment as a teacher; there is no indication that in turn for his services he was to receive advanced instruction, or in fact that he even took up residence with Galigai as a kind of apprentice. The one expense he incurred was half the cost of the book used as the school's roster. He was given three vacations (« andare in villa ») of ten days each year, and he was to have the right to sick leave (without pay) of up to a month and a half. The contract was to have a duration of three years; there was a

²¹ The contract does not specify the kind of *soldo* used in stating the rates; I have assumed that it is a *soldo di piccioli*. Otherwise, if what is involved is the *soldo a oro* Giuliano would have needed so few students to earn his first 12 florins that it would be difficult to understand why the terms would have been expressed in this way.

penalty clause of twenty florins. Finally, once Giuliano left the school he was not to teach in Florence for three months following his departure.

Giuliano was apparently the only other teacher in Galigai's school (they alone shared the cost of the roster, and arrangements for substitute teaching in the event of the illness of one of them takes no account of any other colleagues). Galigai collected the fees from the students and himself paid Giuliano, and from what we know of this arrangement it appears that he did not do badly at all. The difference of 23*s.* between the totals of the two rates was Galigai's minimum profit on each student who took the full program once Giuliano had earned twelve florins (assuming, of course, that the student fee was at least 32*s.*). If in fact a student paid as much as the one florin Bernardo Machiavelli paid for Niccold's instruction in a *scuola d'abbaco* in 1480,²² then Galigai's profit on Giuliano's teaching can only be considered exorbitant: less than 25 percent (32*s.* out of one florin) of the student's fee paid for a complete program would have gone to Giuliano at the higher rate, only about 6 percent (9*s.* out of one florin) at the lower.

We know from the contract that in Galigai's school at any one time there were students in several courses, and that very likely some students entered with advanced standing having already studied the material of the earlier two or three courses at home or at another school. What the total enrollment was at any one time is impossible to say. According to Villani's statistics, there were six schools with an average of about 200 students each, which seems very high even allowing for several sessions with different students. In the anecdote about Maestro Giovanni di Bartolo, the two threatened teachers were in a position to send together fifty students to Giovanni's new school, but these may have been students at just one level of instruction. As for the length of the course, we can assume from the stipulations regarding Giuliano's pay with respect to dropouts that they lasted well beyond fifteen days of instruction; and to judge from the rates all the courses except the first must have had more or less the same duration. It has usually been assumed that this period in a boy's life when he was attending a *scuola d'abbaco* lasted about two years, the difference between the ages when they first took up their studies and when they began their apprenticeship.²³ Nevertheless, on all these matters precise information is still lacking, and it is therefore not possible to go any farther with an economic analysis of a *scuola d'abbaco*.

²² « Ricordo come questo dì 3 di gennaio 1479 io alloggi Nicolò mio figliuolo a Piero Maria maestro d'abaco che gl'insegnassi l'abaco, e d'acordo fumo gli dovessi dare per insegnatura di tutto fiorini uno largo in questo modo, cioè: uno mezo quando entrerà nelle librettine, e un altro mezo fornito gli arà d'insegnare »; BERNARDO MACHIAVELLI, *Libro di ricordi*, ed. Cesare Olschki (Florence, 1954), p. 103.

²³ SAPIORI, *La cultura del mercante*, p. 67, n. 1. Twenty-two months passed between the time Machiavelli entered a *scuola d'abbaco* and the time he took up the study of Latin; but for Landucci it was less than fifteen months after he entered a school that he began his apprenticeship.

* * *

Treaching was the chief professional activity of the *maestri d'abbaco* and their schools, or *botteghe*, must have provided them with their basic income, even though our contract makes it clear that Galigai left at least some of the elementary teaching to an assistant. In addition, however, they could supplement this income by their services as consultants whenever their mathematical knowledge was needed. It was above all in the construction industry where their learning had the most frequent practical application. Maestro Giovanni di Bartolo, of the well-known school of Santa Trinita mentioned at the beginning of this article (whose father was a *muratore*), was called in as a consultant several times by the commission in charge of building the cupola of the cathedral,²⁴ and it is not difficult to imagine why the services of one who could make mathematical calculations might have been indispensable in such a daring project. A more usual and undoubtedly more intellectually modest activity of these *maestri*, one frequently encountered in building accounts of the period, was that of official outside measurer who was engaged after the completion of building projects (or phases thereof) in order to settle accounts between patron and builder. This was especially true in the case of foundations for new buildings inasmuch as founders were paid according to the cubic space prepared. General contracting for building itself was much rarer in Florence; but occasionally builders would undertake the construction of walls on a rate rather than a wage basis, and in these cases too an outsider, invariably a *maestro d'abbaco*, was called in to take final official measurements for purposes of settling accounts. In the building accounts of the Ospedale degli Innocenti, just to take one of the many possible examples, a Maestro Mariano *d'abbaco* appears periodically to take measurements of completed walls as the outside arbiter between the builders and members of the orphanage's building commission. The fees were not large, but these men must have welcomed this supplement to the modest income from their schools.²⁵

Their interest in mathematics, however, went well beyond practical ap-

²⁴ On 30 June 1417 he was paid five florins for « consiglio sopra la Chupola maggiore »; in 1420 he received one florin for his services; and in 1428 he was paid another ten florins « per sua faticha e mercè appiù disengni e modi per lui dimostrati nella Chupola maggiore ». CESARE GUASTI (editor), *La cupola di Santa Maria del Fiore* (Florence, 1857), pp. 19, 26-27, 33.

²⁵ One might think that, given their professional interest in commercial mathematics, these *maestri* might also have earned money on the side as accountants. It is not at all sure, however, that they taught accounting (cf. note 20). Furthermore, in the institutional accounts I am familiar with — for example, those of the Ospedale degli Innocenti — the accountants who are periodically called in to audit the books are always identified as *ragionieri*, not *maestri d'abbaco*; and in fact I cannot associate any of their names with known *maestri*. In Lucca, on the other hand, the *maestri* on the public payroll served at times as *ragionieri* of the *camera* and *computisti* of the commune; BARSANTI, *Il pubblico insegnamento*, p. 56.

plications and the immediate professional needs of teachers of commercial arithmetic. They were also men of learning in their subject and its related fields. The fourteenth century Maestro Antonio de' Mazzinghi da Peretola, teacher and predecessor of Maestro Giovanni di Bartolo, was known to be learned in arithmetic, geometry, astrology, music (all part and parcel of mathematics of the time) and «in edificare, in prospettiva...».²⁶ He had a large library, and so did his successor Maestro Giovanni himself (in his 1427 Catasto declaration he mentions books on astrology). Giovanni's stature as an intellectual was sufficient to gain him a three year appointment at the Florentine Studio from 1424 to 1427, for which he received a stipend of 72 florins a year — handsome perhaps for him, but considerably below the 100 to 300 florins paid to professors of law and medicine.²⁷ His library was dispersed after his death, but even a generation later there seems to have been knowledge of what had happened to the many volumes; it presumably was a central collection for his fellow mathematicians.

Giovanni himself apparently wrote nothing that has survived, but other Florentine *maestri d'abbaco* did and their treatises exist in numerous manuscripts and early editions. Gino Arrighi is doing a remarkable task in publishing or describing a number of them,²⁸ and almost all the earlier commercial arithmetics in the Plimpton collection — both in manuscript and printed — are Florentine.²⁹ The manuscripts were widely circulated and some exist today in a number of variant copies.³⁰ They borrow heavily from one another

²⁶ From the treatise of Maestro Benedetto da Firenze (ca. 1463), excerpted in ARRIGHI, *Il codice L.IV.21*, p. 398. This treatise is the source for much of what we know about Giovanni di Bartolo.

²⁷ GENE A. BRUCKER, *Florence and Its University, 1348-1434*, in «Action and Conviction in Early Modern Europe», ed. T. K. Rabb and J. E. Seigel (Princeton, 1969), p. 231. The median stipend in the arts faculty in 1439 was 32 florins; *ibid.*, p. 236n.

²⁸ These include: PAOLO DELL'ABBACO, *Trattato d'aritmetica* (Pisa, 1964); LEONARDO FIBONACCI, *La pratica di geometra volgarizzata da Cristofano di Gherardo di Dino* (Pisa, 1966); MAESTRO ANTONIO DE' MAZZINGHI, *Trattato di fioretti nella trascelta di Benedetto* (Pisa, 1967); PIERO DELLA FRANCESCA, *Trattato d'abaco* (Pisa, 1970); FILIPPO CALANDRI, *Aritmetica secondo la lezione del Codice 2669 (sec. XV) della Biblioteca Riccardiana di Firenze* (2 vols.; Florence, 1969). In addition, summaries and extracts from other treatises have been published by ARRIGHI: *Il codice L.IV.21; La matematica a Firenze nel Rinascimento: il codice ottoboniano latino 3307 della Biblioteca Apostolica Vaticana*, «Physis», vol. X (1968), pp. 70-82; *Il trattato d'abaco di M^o Benedetto da Firenze del Codice 5570 (sec. XV) della Biblioteca Marciana di Venezia*, «Bollettino della Unione Matematica Italiana», ser. IV, vol. I (1968), pp. 146-50; *Paolo dell'Abaco e Benedetto da Firenze*.

²⁹ See the published catalogue by DAVID EUGENE SMITH, *Rara Arithmetica* (Boston, 1908). One of the Columbia University manuscripts dating from the fourteenth century and presumably Florentine has been described in very great detail by ELIZABETH BUCHANAN COWLEY, *An Italian Mathematical Manuscript*, in «Vassar Medieval Studies», ed. C. F. FISKE (New Haven, 1923), pp. 379-405.

³⁰ The treatise of Maestro Benedetto (ARRIGHI, *Il codice L.IV.21*) exists in copies in Florence, Siena, Rome, Venice, Paris, and New York (the Plimpton Collection at Columbia University).

in the material they handle, and there is sometimes an historical section in which they survey the better known mathematicians from Paolo dell'Abbaco to their own contemporaries — and these are in fact about our only sources for the biographical information we have on these men. Thus it was that Antonio de' Mazzinghi and Giovanni di Bartolo, for example, became almost legendary figures within the ranks of their profession. These *maestri d'abbaco*, in short, not only formed a distinct profession in Florentine society but they also had a strong sense of themselves as a group of intellectuals with common interests. They were not humanists; and, except for Paolo dell'Abbaco, whom both Boccaccio and Matteo Villani praise, they perhaps did not number among the truly great men of their times in the eyes of the more intellectual and the more powerful of their contemporaries. Yet a simpler man like Luca Landucci could put two *maestri* along with two bishops, two musicians, seven artists, a scientist, and Cosimo de' Medici in his list of « nobili e valenti uomini » living in Florence about 1460.³¹ Their professional reputations extended abroad as well, and even Venetians recognized their pre-eminence throughout Italy.³²

Their treatises are primarily books of practical, or applied, mathematics written for the use of merchants in which basic calculations are illustrated with the kinds of practical problems a merchant might well encounter in the everyday conduct of his affairs. Galigai himself published such a treatise entitled *Summa de Arithmetica* (Florence, 1521).³³ It was dedicated to Cardinal Giulio de' Medici (later Clement VII) and it later went through two more editions. The bulk of the work could easily have been a textbook for the kind of course which merchants' sons went through in Galigai's — or any other master's — school. It explains how to perform the basic mathematical operations for a monetary and mensural system based on duodecimals and vigesimal (*soldi* and *denari*); how to make conversions from foreign moneys, weights and measures; how to solve problems involving interest and time, exchange of goods and shares in partnership organizations. The presentation takes the form primarily of exercises involving typical kinds of problems a merchant might be confronted with. In these respects Galigai's book is strictly a commercial and financial arithmetic. Other treatises, how-

³¹ «...maestro Mariano che 'nsegnava l'abbaco; Calandro maestro d'insegnare l'abbaco e uomo molto buono e costumato, che fu mio maestro », LUCA LANDUCCI, *Diario fiorentino dal 1450 al 1516*, ed. Iodoco del Badia (Florence, 1883), p. 3.

³² BEC, *Les marchands écrivains*, p. 388.

³³ In the later editions it is entitled *Pratica d'arithmetica*. I have used the Florentine edition of 1552 in the Plimpton collection at Columbia University. See SMITH, *Rara Arithmetica*, p. 132; B. BONCOMPAGNI (editor), *Bullettino di bibliografia e di storia delle scienze matematiche e fisiche*, vol. VII (1874), pp. 484-87; vol. XIII (1880), pp. 248-50; and PIETRO RICCARDI, *Biblioteca matematica italiana dalla origine della stampa ai primi anni del secolo XIX*, vol. I (Modena, 1893), cols. 499-502 (also vol. II, *correzioni ed aggiunte*, ser. I, II and V).

ever, contained in addition sections devoted to commercial customs, with compilations of international exchange rates and measures. Even the very first of these Italian treatises on arithmetic, Fibonacci's *Liber abbaci* of 1202, had included, albeit incidentally, some nonmathematical material in the form of useful information on trade; and such content became much more important in many of the later treatises. It was, after all, in a general treatise on mathematics (which incidentally had less of a practical orientation than most) that another Tuscan Luca Pacioli included the famous section on double entry bookkeeping, the first published discussion of the subject.³⁴ To the extent that the mathematical treatise tended to become a compendium of practical information needed by the international merchant-banker, the genre blends into the much better known one of merchants' manuals. Merchants' manuals often contain arithmetic tables to facilitate the rapid calculations of exchange values, interest payments, and conversions of foreign money, weights, and measures; but they of course were handbooks written by merchants for rapid consultation, whereas the mathematical treatise had a clear didactic purpose in its presentation of numerous problems and in its explanation of arithmetic computations.³⁵

Many of these treatises go beyond purely practical considerations to enter the realm of higher mathematics with discussions of plane and solid geometry and algebra. The last four books of Galigai's treatise, for example, are dedicated entirely to problems of algebra; and in fact his book is probably most notable for its quality as a resumé of the state of knowledge of the subject at the time.³⁶ Nevertheless, for the most part these men were more interested in exercises than demonstrations and enunciations of rules, and they continue to state problems in mercantile terms even when it is clear that such problems have no relevance to mercantile practice. Their attempts to advance beyond

³⁴ There are, however very few references to awareness of accounting problems in earlier mathematical treatises; see the scattered remarks in MELIS, *Storia della ragioneria*, pp. 585-97.

³⁵ A survey and general description of these treatises can be found in DAVID EUGENE SMITH, *History of Mathematics* (2 vols.; Boston, 1923); ID., *The Influence of the Mathematical Works of the Fifteenth Century*; and FANFANI, *La préparation intellectuelle*, pp. 334-42; cf. notes 28 and 29 above. On merchant manuals and their genesis, see SAPORI, *La cultura del mercante*, pp. 78-87; FREDERIC C. LANE, *Manuali di mercatura e proutuari di informazioni pratiche*, in «Zibaldone da Canal: manoscritto mercantile del secolo XIV», ed. Alfredo Stussi (Venice, 1967), pp. xlvii-lviii; and especially for the most recent bibliography on the subject, ROBERT S. LOPEZ, *Un texte inédit: le plus ancien manuel italien de technique commerciale*, «Revue historique», CCXLIII (1970), pp. 67-76.

³⁶ For comment on the quality of Galigai's knowledge of mathematics, see MORITZ CANTOR, *Vorlesungen über Geschichte der Mathematik*, vol. II (reprint: New York, 1965), p. 481; GUGLIELMO LIBRI, *Histoire des sciences mathématiques en Italie, depuis la Renaissance des lettres*, vol. III (Halle, 1865), pp. 145-46. See also LOUIS CHARLES KARPINSKI, *An Italian Algebra of the Fifteenth Century*, «Biblioteca mathematica», vol. XI (1911), pp. 209-19.

didactic presentation to purely theoretical considerations are generally unsuccessful, and in the overall history of mathematics they are more distinguished by their publications record than by their mathematical genius. In this period mathematics was closely tied to the needs for practical application of methods of computation; and although the merchants and bankers of Italy were well served by the mathematicians of their age their demands were limited by the very nature of business practice. A new stimulus to a higher mathematics came only later in the sixteenth century very much as a result of improved techniques in navigation, surveying, cartography, horology, gunnery and fortification, all of which required more complex computations; and in fact in England, for example, a new breed of mathematicians grew up among the practitioners of these new rapidly improving skills. Moreover, to the extent that the practical aspects of these new technologies associated with the physical expansion of Europe and the radical innovation in warfare affected the vital interests of statesmen, the patronage of mathematics in the sixteenth century shifted well beyond business circles and the limited practical needs of merchants and bankers.³⁷

Whatever the quality of Italian mathematical treatises from the point of view of the history of mathematics, their very quantity is a little noted fact in the history of Renaissance culture. There is no guide at all to the manuscripts; but Riccardi's bibliography of printed books in Italy dealing with mathematics is an impressive monument to the popularity of the subject. Altogether according to his statistical analysis there are 214 incunabula and 1527 more publications in the sixteenth century. Although the largest single category of incunabula is astronomy (and astrology), a sizeable number of them can properly be described as commercial arithmetics. Their publication attests to a very large reading public, yet this is a cultural phenomenon which in itself has never been investigated. Historians of mathematics tend to dismiss them, and for the business historians they are eclipsed by the merchants' manuals; but the very existence of so many mathematical treatises — some of the printed ones going through many editions — demands an explanation. They remain one of the great deposits of documents still to be explored by historians of the culture of Renaissance Italy.

The readers of these treatises must have been largely merchants, and this was probably true of the earlier manuscripts as well. Some of these are embellished with patrician coats-of-arms, illuminated initials, and numerous illustrations, and at least one has passages cast in poetry³⁸ — all of which suggests that we are dealing here with a body of literary production that

³⁷ E. G. R. TAYLOR, *The Mathematical Practitioners of Tudor and Stuart England* (Cambridge, 1954), pp. 9-10.

³⁸ FANFANI, *La préparation intellectuelle*, pp. 340-42; examples of the illustrative materials accompanying the manuscripts can be found in the various publications of Arrighi.

deserves to be regarded as a genre in its own right, of particular importance for an understanding not so much of the history of mathematics as of the mercantile mentality of Renaissance Italy. They were written for merchants — in his *Treatato d'abbaco*, only recently published, Piero della Francesca says so explicitly — and they assume a merchant civilization. They show no concern for moral problems about the conduct of business nor doubts about the social status of the businessman.³⁹ They more than suggest, on the other hand, that the Italian merchant — and above all the Florentine merchant — found a fascination in mathematics that went well beyond practical applications. The fact that the treatises more often than not present their more erudite material in the form of problems, and that even when those problems go beyond any practical application they are nevertheless couched in practical terms familiar to the merchant (or in the case of geometry, to the builder) would seem to indicate that for many a merchant working out a mathematical problem had its own satisfaction. One can easily imagine a Florentine merchant in what moments of leisure he might have had fascinated by some utterly improbable mathematical problem involving, for example, capital, time and interest, or division of profits by a partnership, of the kind he might find in a mathematical treatise, and being completely satisfied merely because such mental activity somehow sharpened his wits.⁴⁰

We have a long way to go before we can say we have penetrated the minds of Florentine merchants, and along the way we are going to have to understand their most characteristic expressions — their extraordinary penchant for writing everything down, from petty expenses to the history of their city; their passion for keeping their personal arithmetical record straight with the symmetry of double-entry bookkeeping; their fascination with the purely mathematical problems of these treatises, and finally their taste for perspective and the mathematical organization of space in art and architecture — all are part of a single intellectual whole with a strong mathematical flavor. And all are distinctively Florentine.⁴¹ These merchants were, after all, the products of their education, and that more often than not had culminated in a *scuola d'abbaco*.

³⁹ In these respects the commercial arithmetics of sixteenth century France reflect a very different kind of society; NATALIE ZEMON DAVIS, *Sixteenth-Century French Arithmetics on the Business Life* « Journal of the History of Ideas », vol. XXI (1960), pp. 18-48.

⁴⁰ « Ancora si loda molto l'aresmetrica, cioè l'abacho in detta età, perchè fa l'animo atto et pronto a esaminare le cose sottili »; GIOVANNI RUCCELLAI, *Il zibaldone quaresimale*, ed. Alessandro Perosa (London, 1960), p. 14.

⁴¹ For instance, such matters are not even touched on in the analysis of BEC, *Les marchands écrivains*.

