Some Econometric Problems in the Standard of Living Controversy

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In his "A Theory of Economic History", Sir John Hicks expressed surprise at the fact that during the Industrial Revolution wages failed to rise for a long time. "It is the lag of wages behind industrialization which is the thing that has to be explained", noted Hicks.[1] While Hicks finds himself in comforting company in making this statement — one only has to think of J. S. Mill who did for classical economics what Sir John has done for neoclassical economics[2] — it should be kept in mind that until the present day the controversy on the issue has not been terminated.[3]

To be sure, the debate on whether real wages rose during the Industrial Revolution is not identical with the debate on what happened to the

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[1] J.R. Hicks, *A Theory of Economic History*, Oxford: Oxford University Press, 1969, pp. 148-49.

[2] Recall Mill's famous statement in 1848: « Hitherto [1848] it is questionable if all the mechanical inventions yet made lightened the day's toil of any human being. They have enabled a greater population to live the same life of drudgery and imprisonment, and an increased number of manufacturers and others to make fortunes. They have increased the comforts of the middle classes. But they have not yet begun to affect those great changes in human destiny, which it is in their nature and in their futurity to accomplish. Only when, in addition to just institutions, the increase of mankind shall be under the deliberate guidance of judicious foresight, can the conquests made from the powers of nature by the intellect and energy of scientific discoverers become the common property of the species, and the means of improving and elevating the universal lot ». John Stuart Mill, *Principles of Political Economy*, ed. W. J. Ashley, London: Longmans, Green and Co., 1929, p. 751.

[3] Some highlights of the debate have been reissued recently with a new introduction in Arthur J. Taylor, *The Standard of Living in the Industrial Revolution*, London: Methuen, 1975.

standard of living. The latter is in principle untractable as it is cast in terms of utility, not consumption, and includes accordingly nonmarketed and nonpecuniary items. One can have opinions about what happened, but one cannot test hypotheses. Neither better theory nor better econometric techniques can ever resolve a question about standards of living in the sense of the overall quality of life.

The question of wages is interesting, however, not only because of its bearing upon the standard of living. The behaviour of wages can cast, for example, light on the sources from which factory workers were recruited, or the changes in the requirements of human capital used in the early stages of modern industry. Of equal interest is the behaviour of real wages in view of the theory that profits were the primary propellant fuel in the engine of capital accumulation. Rising wages would, other things equal, slow down the modernization process.[4]

The exists a large number of time series of various quality for both wages and prices in England as well as some other European countries.[5] There are two types of statistical problems with the analysis of the data. The first type are the traditional difficulties encountered by economists concerned with aggregation. The construction of price indices covering long periods contains severe sources of biases, as is well known. One can overcome such problems sometimes by deliberately biasing the index against one's conclusion. For instance, if the null hypothesis is that prices rose, one may choose the weights from the beginning of the period (Paasche), in order to bias the index against the hypothesis. This is one way of minimizing the chance of accepting the hypothesis when it is false. Less serious appears the claim that since there was considerable diversity in regional experience, detailed studies are necessary before any sound generalizations can be made.[6] Such arguments, in our view, tend to be rather barren: there can be no doubt that even in the most prosperous periods of growth, there will be areas and industries which do not share the general trend. The fact that wages are distributed around some kind of mean reinforces the notion that an index number may be interpreted as an estimate of a population mean. With labour and commodity markets becoming gradually more efficient, and factors and goods more mobile, it becomes

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[4] For a full exposition of this argument, see JOEL MOKYR, "Growing Up and the Industrial Revolution in Europe", *Explorations in Economic History*, 13, (1976), pp. 371-96.

[5] For a catalogue of the various indices in England, cf. M. W. FLINN, "Trends in Real Wage, 1750-1850", *Economic History Review*, 2nd Series, XXVII (1974), 393-413. Some data on continental economies during various stages of the industrialization process are provided in BRIAN R. MITCHELL, *European Historical Statistics*, London: Macmillan, 1975, pp. 184-190, 742-746.

[6] T. R. GOURVISH, "The Cost of Living in Glasgow in the Early Nineteenth Century", *Economic History Review*, 2nd series, XXIV (1972), pp. 65-80.

increasingly unlikely that very significant discrepancies could persist for long periods, i.e., the dispersion around the population mean will decrease.

A second type of difficulty exists, however, in the real wage controversy due to the inherent difficulties involved in time series. The severe fluctuations which perturbed the European economies between 1780 and 1850 imply that some time series — and especially prices — reverse their direction with minor changes in the choice of starting and end points. Almost all series display large variances around their trends, which have caused some historians to try to smooth the series by discarding the observations in the middle, or by using moving averages. A further difficulty with using statistical inference in time series is the existence of autocorrelation, i.e., the deviations from the trend are not independent from each other. The nature of this problem will be discussed below.

The era of the Napoleonic Wars occupies a special role in the history of real wages in Britain. It seems to have been a generally accepted fact that real wages declined during the period 1793-1815. To be more precise a large part of the secular stability of the real wage over the period 1780-1850 can be attributed to the decline of real wages during the Napoleonic Wars, which recovered slowly after Waterloo, reaching its 1780 level at some point between 1830 and 1850, depending on which index one chooses. Why it took wages so long to recover is of course an unanswered question.[8] But is this time pattern an undisputed fact in itself? In his survey paper, Flinn maintained that the available wage and price series do not demonstrate a significant divergence during the war years, thus challenging the traditional conclusion about real wages during the Napoleonic Wars. Flinn's technique was simple indeed, comparing five-yearly averages of 1790-94 and 1810-14.

On this basis he was subsequently challenged by T. R. Gourvish, who argued that more rigorous statistical techniques applied to the same data refute Flinn's conclusion that wages kept pace with prices during the

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[7] See for instance the data collected for London artisans by Rufus Tucker which bear this out most strikingly. R. TUCKER, "Real Wages of Artisans in London, 1729-1935", *Journal of the American Statistical Association* 31 (1936), reprinted in Taylor, ed., *The Standard of Living*, pp. 21-35. For other statements in the same spirit, see for example, Arthur Gayer, W. W. ROSTOW, and ANNA J. SCHWARTZ, *Growth and Fluctuations of the British Economy, 1790-1850*, Oxford: Oxford University Press, 1953, p. 657. PHYLLIS DEANE, *The First Industrial Revolution*, Cambridge at the University Press, 1969, p. 245. PETER MATHIAS, *The First Industrial Nation*, London: Methuen, 1969, p. 46.

[8] In spite of the vast dissimilarity in circumstances, a very similar thing happened to American real wages in the civil war. During the war years, inflation eroded the purchasing power of wages, which is not surprising. What is surprising — and indeed hitherto unexplained — is the failure of real wages to recover their 1860 levels until much later (1883). See STANLEY LEBERGOTT, *Manpower in Economic Growth*, New York: McGraw-Hill, 1964, p. 258.

Napoleonic Wars.[9] Gourvish's comments are a good example of the abuse of econometrics in historical research, so his note will be dealt with in some more detail. Gourvish used least squares to fit trend lines to logs of five-yearly moving averages. The coefficients of the time variable — to be interpreted as compound growth rates — tended to be somewhat larger than those calculated by Flinn. From a point of view of rigorous statistical inference, however, what one has to do is to test the hypothesis that Flinn's "crude" growth rate is different from the least squares trend growth rate calculated by Gourvish.

For purpose of illustration, consider the Schumpeter-Gilboy consumers prices index. For the period 1788/92 - 1809/15 Flinn calculated the percentage increase in the Schumpeter index as 69.4%. Let β be the instantaneous rate of growth of the function $e^{\beta\tau}$. Using the percentage increase calculated by Flinn the implied instantaneous growth rate is $\beta = \log_e 1.694/23 = .0229$. Now suppose we wish to test the hypothesis that this is the "true" growth rate, i.e., the null hypothesis $\beta = .0229$. For this purpose we specify that the five-year moving averages of prices centred on year τ satisfied the relation

$$(1)\quad Y_\tau = A\, e^{\beta\tau} U_\tau \qquad \tau = 1, \ldots, T.$$

which may be written as

$$(2)\quad Y_\tau = \alpha + \beta\tau + U_\tau \qquad \tau = 1, \ldots, T.$$

where $Y = \log^e Y$, $\alpha = \log^e A$ and $U = \log^e U$. Note that we specify that disturbance, U, in (1) is multiplicative in order that (2) will have the required additive disturbance, U. If the assumption of the classical normal linear regression model are appropriate for (2), then the 100 $(1 - \varepsilon)$ per cent confidence interval for β is given by

$$(3)\quad b + t_{\varepsilon/2} S_b$$

where b is the least squares estimate of β, S_b is the estimated standard error of b and $t_{\varepsilon/2}$ in the upper $\varepsilon/2$ significant point of the t — distribution with t - 2 degrees of freedom where T is the sample size. Taking five-year moving averages of the Schumpeter index for the period 1788/92 — 1811/1815 and fitting a log linear trend line by least squares gives $b = .0260$ and $S_b = .00166$. The upper .025 significant point of the t - distribution with 22 degrees of freedom is $t_{.025} = 2.074$ and hence the lower and upper limits of the 95 per cent confidence interval are .0226 and .0295, respectively. To test the hypothesis $\beta = .0229$ at the 5 per cent level of significance we see if the 95 per cent confidence interval

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[9] T. R. Gourvish, "Flinn and Real Wage Trends in Britain, 1750-1850: A Comment". *Economic History Review*, 2nd Series, XXIX (1976), pp. 143-145.

contains the hypothesized value. Since the interval includes the value .0229 the hypothesis is accepted. Of course, the same procedure can be used to test the hypothesis at any level of significance. Gourvish notes that when the trend lines are fitted to raw data the differences between Flinn's and his own growth rates are even larger. Taking the Schumpeter index for the period 1790-1813 and fitting a log linear trend line gives $b = .0274$ and $S_b = .00299$. In this case the lower and upper limits of the 95 cent confidence interval are .0212 and .0336, respectively, which also include the hypothesized value.

The above procedure may also be used in connection with the other price indexes. We restrict our attention to indices in Mitchell and Deane;[10] Schumpeter; Gayer, Rostow and Schwartz; Rousseaux. The results for these series are given in Table 1.[11] For each index the 95 per cent confidence interval calculated from least squares regressions on either moving average data or raw data include the instantaneous growth rate derived from Flinn's percentages except in the case of moving averages for 1813-24 based on the Gayer, Rostow and Schwartz data. In this light Flinn's conclusion that Gourvish's results are comfortingly close to his own figures appears to be correct in the sense that growth rates estimated by least squares are not significantly different at the 5 per cent level from the rates implied by Flinn's calculations.

Flinn's conclusions tend to be further strengthend due to the presence of autocorrelated disturbance. If in fact the disturbances are positively autocorrelated, then the usual least squares formulae for the sampling variances of the regression coefficients are likely to give a serious underestimate of these variances and hence of the width of the confidence intervals. Of course, the wider the confidence intervals the less likely that Flinn's and Gourvish's results are significantly different. In Table 1 we report the Durbin-Watson "d" statistic for each regression. Since the regressions are pure trend regressions the true critical value of the "d" statistic is well approximated by the upper bound on the true critical value.[12] With one explanatory variable the upper bound at the 5 per cent significance level is 1.331 ($T=12$) and 1.446 ($T=24$).[13] The "d" test rejects the hypothesis of indipendence in favour of positive autocorrelation in the

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[10] These are the most widely used series. Other series, such as Tucker's cost of living index or Silberling's wholesale price index are strongly correlated with these series.

[11] Gayer, Rostow and Schwartz not included for 1788/92-1811/15 since we were not able to reproduce Gourvish's regression results.

[12] J. Durbin, "An Alternative to the Bounds Test for Testing for Serial Correlation in Least Squares Regression", *Econometrica* 38 (1970), 422-429.

[13] N. E. Savin and K. White, "The Durbin-Watson Test for Serial Correlation with Extreme Sample Sizes or Many Regressors", *Econometrica* 45 (1977), pp. 1989-96.

TABLE 1

LEAST SQUARES ESTIMATES: 95% CONFIDENCE INTERVAL FOR THE GROWTH RATE OF PRICES *

Index/Period	$b \times 10^{-1}$	$S_b \times 10^{-2}$	$b \pm t_{.025} S_b$		H_o **	d ***
			Five-Year Moving Averages			
Schumpeter 1788/92 - 1811/15	.260	.166	.0266	.0295	.0229	.318
Gayer, Rostow and Schwartz 1811/15 - 1822/6	—.421	.201	—.0466	—.0378	—.0333	1.035
Rousseaux 1811/15 - 1822/6	—.443	.212	—.0490	—.0396	—.0389	.789
			Raw Data			
Schumpeter 1790 - 1813	.274	.299	.0212	.0336	.0229	1.006
Gayer, Rostow and Schwartz 1813 - 1824	—.480	798	—.0658	—.0302	—.0333	1.134
Rousseaux 1813 - 1824	—.499	.642	—.0642	—.0356	—.0389	1.341

* The upper and lower limits of the 95% confidence interval are $b \pm t_{.025} S_b$ where b is the least squares estimate of the growth rate, S_b is the estimated standard error and $t_{.025}$ is the upper .025 significance point of the t distribution with T-2 degrees of freedom.

** The instantaneous growth rate derived from Flinn's arithmetic percentages.

*** Durbin-Watson statistic. With one explanatory variable the upper bound on the .05 critical value of d is 1.331 (T = 12) and 1.446 (T = 24). The source is Savin and White cited in footnote 6.

case of all regression except for the regression based on the raw Rousseaux index.

To explore the impact of positive autocorrelation on the least squares estimates of equation (2) we reestimated this equation by the method of maximum likelihood [14] (ML) under the assumption that the disturbance, U , follow a first-order autoregressive process. The ML estimates and approximate 95 per cent confidence intervals are reported in Table 2. The reestimated growth rates are roughly the same as the least squares growth rates. On the other hand, the reestimated standard errors are substantially

[14] ML estimation was performed using a computer programme written by K. WHITE, "SHAZAM, An Econometrics Computer Program", Department of Economics, University of British Columbia (1976). See K. WHITE, "A General Computer Program for Econometric Methods SHAZAM", *Econometrica* 46 (1978), pp. 239-240. This programme incorporates the grid search procedure described in P. J. DHRYMES, *Distributed Lags: Problems of Estimation and Formulation* (1971), 64-70. Other indices (such as Tucker's London index) are so strongly correlated with the indices used that it was unnecessary to experiment on all indices catalogued by Flinn.

larger than those produced by least squares. Thus, when positive autocorrelation is taken into account the resulting confidence intervals are considerably wider. For example, using moving averages based on the Schumpeter data the ML procedure increases the confidence interval from 2.26% — 2.95% to 1.93% — 3.20%. The results correcting for autocorrelation should be viewed with caution since large sample methods have been applied to comparatively small samples and since the disturbances may obey an autoregressive process which is higher than first-order. Nevertheless, it is apparent that inferences based on least squares estimates may be misleading in the present context.

The least squares regression on the moving average data produces a different growth rate than the least squares regression on the raw data. This in not surprising since if relation (1) hold for the moving averages it does not hold for the raw data and vice versa. Hence the results of hypothesis testing may depend on which regressions are chosen although this was not the case in the present context. Gourvish selected the re-

TABLE 2

MAXIMUM LIKELIHOOD ESTIMATES (FIRST-ORDER AUTOCORRELATION) APPROXIMATE 95% CONFIDENCE INTERVALS FOR GROWTH RATES OF PRICES *

Index/Period	$b \times 10^{-1}$	$S_b \times 10^{-2}$	$b \pm 1.96\, S_b$		H_o**	d **
			Five Year Moving Averages			
Schumpeter 1788/92 - 1811/15	.257	.325	—.0193	.0320	—.0229	.745
Goyer, Rostow and Schwartz 1811/15 - 1822/6	—.408	.278	—.0486	—.0356	—.0333	.958
Rousseaux 1811/15 - 1822/6	—.419	.340	—.0463	—.0354	—.0389	.876
			Raw Data			
Schumpeter 1790 - 1813	.278	.447	.0190	.0365	.0229	1.379
Gayer, Rostow and Schwartz 1813 - 1824	—.475	1.037	—.0678	—.0271	—.0333	1.221
Rousseaux 1813 - 1824	—.493	.775	—.0639	—.0347	—.0389	1.446

* The upper and lower limits of the approximate 95% confidence interval are $b \pm 1.96\, S_b$ where b is the ML estimate of the growth rate, S_b is the estimated standard error and 1.96 is the upper .025 significance point of the standard normal distribution. The estimate, S_b is calculated using T-2 as the divisor of the sum of squared ML residuals in order to permit a straightforward comparison with usual squares formula for S_b.

** See footnotes Table 1.

gressions on the moving average data in order to minimize the impact of the extreme observations on the least squares estimate of the growth rate. An alternative procedure is to remove the extreme observations from the sample before running regressions on the raw data. Since using moving averages involves a loss of information — although the loss is not as obvious as when observations are discarded — it is a dubious procedure unless it has a firm basis in economic theory. On statistical grounds we would choose the regressions on the raw data since these regressions appear to have less highly autocorrelated disturbances as measured by the "d" statistic. It is worth remarking that if the raw data consists of statistically independent observations, then the moving averages are statistically dependent.

To return to the issue of real wages: the behaviour of real wages during the Napoleonic Wars and its aftermath hinges on whether the trends for nominal wages are different than the trends for consumer prices. Of the four nominal wage indices cited by Flinn (two indices of Phelps-Brown and Hopkins, Wood, and Tucker - omitting Kuczynski), three have growth rates which fall in the 95 per cent confidence interval based on least squares estimates of the growth rate of the Schumpeter index, while the Tucker index indicates a fall in real wages.[15] The inference drawn that at least some evidence seems to indicate a fall in real wages may be misleading, since the confidence interval based on Maximum Likelihood estimates contains all four wage indices.[16] For the aftermath period (1815-22), the Phelps-Brown-Hopkins and Wood wage figures show a rise in real wages which is significant at the 5 per cent level for both least squares and the ML estimates.

One might object that the quality of the wage data does not allow any valid inferences concerning the standard of living. But even given a suitable real wage series it does not follow that the controversy over real wage trends can be resolved. From the point of view of statistical inference a sample of real wages is one possible outcome of a conceptual experiment. The experiment consists of repeating the Industrial Revolution from 1750 to 1850 a

[15] It should be noted that we have ignored the fact that the calculated wage series growth rates are also random variables, i.e., are estimates and not the "true" growth rates. The usual F test of this hypothesis assumes that the observations on wages are independent of each other, that the same holds for observation on prices, that the observations of wages are independent of those of prices, and that the variances of wages and prices and the same. Since these assumptions do not appear to be satisfied we have not performed an F test of the hypothesis of equal growth rates. Of course, large sample techniques which take account of these difficulties are available, but the sample sizes employed here are too small to warrant the application of such techniques (since the periods under consideration are short). Thus, these considerations reinforce our conclusion that statistical inference is unlikely to decide the real-wage controversy.

[16] If we use the price index provided by Tucker himself, however, the trend of real wages is downward.

great many times. Of course, this conceptual experiment cannot be performed, but it is not senseless to contemplate it. Indeed an experiment of this kind is the basis for all econometric studies involving time series data. It supplies the intuitive background for the notion that the least squares estimate of the growth rate has a probability distribution. Presumably what we wish to find out is whether the process we describe as the Industrial Revolution tended to raise real wages and perhaps the standards of living. For the standard of living debate it is the secular trend of the period which is of interest and not the short-run random shocks due, for example, to the weather. A wide confidence interval for the growth rate of real wages implies that a number of diverse hypotheses are consistent with the data. Whether we can resolve the standard of living debate depends on the nature of the random shocks. Since these shocks appear to have been substantial in magnitude it may not be possible to estimate with much precision the systematic effect of the Industrial Revolution on real wages from data produced by a single repetition of the experiment.