The Complex Nature of the Vintage Violin Market and Its Luthiers: An Economic History (16^{th}-21^{st} Century)*

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ABSTRACT

The violin was created in northern Italy during the 16^{th} century and became an important contributor to classical music. In this article, we examine the influence network associated with vintage violins (those made from 1590 to 1875) and their market (from 1590 to 2022). Our focus is on violins attributed to the top 30 violin makers (luthiers); their rank is determined by the highest transaction price ever received for one of their violins. Using the Hawkes self- and mutual-exciting point process and Bayesian estimation techniques, we find that a luthier's past transactions positively influence the future transactions of other luthiers. The strongest influence, however, is the positive impact of a luthier's past transactions on his future transactions. The top three luthiers from 1590 to 1875 in terms of total influence are Antonio Stradivari (Cremona, Italy), Giovanni Guadagnini (Turin, Italy), and Jean-Baptiste Vuillaume (Paris, France). From 1876 to 2022 the top three list changes slightly with Giuseppe Guarneri "del Gesù" (Cremona, Italy) replacing Guadagnini. Our results reflect some of the current beliefs of many violin appraisers and illustrate the theoretical and practical importance of networks in understanding economic history.

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1. Introduction

Music has been embedded in human culture for thousands of years, and it is an important ingredient in the development and maintenance of social organization and group cohesion (e.g., Hill, et al., 2009; Trehub, et al., 2015). Music contributes to these societal attributes by providing entertainment, supporting rituals, and enabling a medium of communication. Although there are several broad categories of musical instruments, those characterized by bows, strings, and resonators (sound boxes) have played an important part in most cultures. They are currently used by symphony orchestras, chamber ensembles, and solo performers in numerous cultures and contexts (Rossing, 2010).

String instruments, according to Montegu (2017), were likely first made during the Mesolithic period (10000-8000 BCE) and almost certainly made in the Neolithic period (8000-3000 BCE). The latter period witnessed the change from a hunter-gatherer social structure to one relying on agriculture and the domestication of animals. Historians posit that this reliance was the reason that the Sumerian people migrated to southern Mesopotamia (currently located in modern day Iraq and Kuwait) around 4000 BCE. Here they could take advantage of the Tigris and Euphrates Rivers' abundant water supply and rich soil. As a result, this region is often referred to as the "Cradle of Civilization" and eventually became the home of a dozen Sumerian city-states with each having its own theocratic leader. In addition to providing sustenance for themselves, city-state residents engaged in trade with other Mesopotamian city-states and beyond. The Sumerians made many important cultural and technical contributions, including inventing writing, extending mathematical knowledge, and constructing percussion, wind, and string musical instruments. The string instruments included harp, lute, and lyre.

The violin, a member of the lute family, appeared toward the end of the European Renaissance (1400-1600), a period that witnessed the rebirth of music, art, literature, and philosophy. Schoenbaum (2013, p. xviii) suggests that the violin's "…impact on Western culture has been as radical as the printing press and the steam engine." Currently children often learn to play the violin as part of their educational experiences, adults play the instrument as a satisfying hobby, and professional violinists perform for audiences. Some vintage violins are acquired by individuals not only for their musical and historical significance but also as investments. They are also viewed as art and are sometimes donated to museums.

The purpose of this article is twofold. First, it is to provide a brief survey of the origins and development of the violin. Currently, its ancestry can be traced back to the 4^{th} millennium BCE. Second, it is to model, quantify, and better understand the transaction complexity of the market for collectable investment-grade vintage violins (those made prior to 1876) from their beginning in the 16^{th} century until the early 21^{st} century. To accomplish this latter task, we focus on the top 30 luthiers as defined by the highest purchase price for a specific luthier's violin.[1]

Thus, we investigate a group of assets that can increase in number only because of an extraordinary discovery and may decrease due to physical damage or some other unforeseen misfortune, such as incorrectly identifying the maker. We use the Hawkes (1971, 2018) self- and mutual-exciting point process in conjunction with the Bayesian estimation procedures developed by Linderman and Adams (2014, 2015) to quantify the transaction complexity of the violins associated with these luthiers.[2] The statistical results are interpreted using the concepts of Granger (1969, 2004) causality, which

[1] A luthier is an individual who makes, repairs, and copies musical instruments that are members of the lute family. Examples of the modern members of this family include violins, guitars, mandolins, banjos, and cellos.

[2] Bouchaud (2010) illustrates that a network depicts a market's inner dynamics because its framework allows feedback effects. Kuhlmann (2014) argues that a complex system approach is useful to understand the internal workings of a financial market. As evinced by Easley and Kleinberg (2010) and Soramäki and Cook (2016), among others, networks have become a useful approach to understand the interactions associated within a market or among many markets.

can be interpreted in the context of prediction or influence (Hlaváčková-Schindler, et al., 2024). As a result, we provide a new historical perspective to the vintage violin transaction network dynamics.

We show that the market for investment-grade vintage violins is interconnected, as evinced by their transaction history spanning nearly half a millennium. Our focus, however, is not on violins themselves, but on the top 30 luthiers who made them, because, as Muñoz (2020) points out, vintage violins (those roughly 150 years or older) are not only typically illiquid investments but are also usually identified by their luthier. We find that the transactions of a top 30 luthier's violins influence the future transactions of his own violins as well as the violins of the other top 30 luthiers. Considering these luthiers as a group, self-influence is the most important phenomenon, with influencing other luthiers and being influenced by them having relatively smaller impacts. The large self-influence phenomenon is attributed to a handful of well-known luthiers such as Antonio Stradivari and Giuseppe Guarneri "del Gesù." The influences for many other makers are small by comparison.

The remainder of this article is divided into five main sections and a short appendix containing summary data. Section 2 summarizes the evolutionary history of vintage violins and their roles as cultural and alternative investment assets. The Hawkes statistical approach used to determine the complex sequential interactions among the luthiers and their violins is described in Section 3. Section 4 discusses and summarizes the violin transaction data used in the study. The empirical research results and possible reasons for them are presented in Section 5. Section 6 offers concluding remarks.

2. A Brief History of the Violin and Its Investment Potential

2.1 Ancestors and Evolution of the Violin

According to the Hornbostel and Sachs (1914/1961) classification system, which is still used today, there are five major categories of musical instruments.[3] Violins fall into the Chordophone category (HS: 3), which has a subcategory dubbed Composite Chordophone (HS: 32). This subcategory contains instruments that have string and resonator components that cannot be separated. This subcategory is further broken down into five families: bows, harps, lyres, lutes, and zithers. Violins are part of the lute family (HS: 321) and are considered as neck-box lutes that require a bow to play (HS: 321-322-71).

Recent archeological excavations in Uruk, a Sumerian city-state (35th-32nd century BCE), provide important evidence concerning the violin's ancestry. Dumbrill (2010, 2011) reports that one of the excavations unearthed a "cylindrical seal" depicting a woman, sitting in a small boat holding a musical instrument that appears to be a lute of some kind. This seal is on display at the British Museum (reference number 142632); Dumbrill (2010, 2011) contends that this historical artifact depicts the violin's most ancient lute ancestor. Musical archaeologists generally accept this claim but leave open the possibility that the lute may have been made by an earlier semitic tribe.

The Byzantine lyra, a member of the lute family, was developed in the Eastern Roman Empire during the 9th century. It is believed to be the oldest direct ancestor of the violin because it combines the sounds of vibrating strings that are parallel to its body with a resonator to create a single musical instrument. This musical instrument was followed by the rebec (11th century), which found its way to Spain as result of significant Muslim immigration. The French vielle (12th century) followed, as did the Italian lira da Braccio (15th century). The latter is the immediate ancestor of the violin.

[3] Lee (2020) provides a history of the Hornbostel and Sachs (HS) classification system for musical instruments and concludes that it is a "…monument of music and organology history…" and has become "…a central classification scheme for curating and studying instruments, as well as playing a central role in musical instrument research and practice (p. 88)." The HS system is based on how musical instruments make their sounds. It denotes this classification system by using numbers and can be modified if the need arises. Modifications, however, must be approved by the Musical Instruments Museums Online (MIMO) Working Group for Classifications and Thesauri. The newest revision is available at: https://en.wikipedia.org/wiki/Hornbostel%E2%80%93Sachs.

The violin was created in the early 1500s. The first physical record of its existence is believed to be a painting by Gaudenzio Ferrari completed circa 1530. Named "La Madonna degli aranci" (Our Lady of Oranges), it is currently an altarpiece on display at the Church of St. Christoforo in Vercelli, Italy. The painting depicts the Virgin Mary surrounded by angels who hold string instruments shaped like violins. Whether these musical instruments were based on imagination or actual violins is unknown, but the accuracy of their depiction strongly suggests the latter.

Two of the earliest known makers (luthiers) of the violin are Andrea Amati (b. 1505-d. 1577) of Cremona and Gasparo di Bertolotti "de Salo" (b. 1542-d. 1609) of Brescia. Cremona and Brescia are 48 kilometers (30 miles) apart and are in Lombardy, which is currently part of the Alpine-Po region of modern northern Italy. Lombardy gave the luthiers' workshops easy access to maple and spruce (the preferred woods for violins) and to sawmills capable of providing the wood planks needed for violin construction (Schoembaum, 2013, p. 12). It also enabled them to exploit the extant water transportation systems (river, canal, and other waterworks) to ship their violins to their customers located in present-day Italy, Europe (especially the Mediterranean area), as well as the Middle and Far East. This transportation network was instrumental in sharing various aspects of culture throughout the region, including artwork, philosophy, and musical instrument innovations.

2.2 Violin Design

Since its beginning, the physical characteristics of the violin have remained relatively stable, but adaptations were made to improve the instrument's sound and playability. For example, the early versions are characterized by narrower bodies and more rounded corners than today's shape (Chitwood, 2014). A "C-shaped" hole was created on each side of the bridge to enhance the instrument's acoustics. During the late 16th and early 17th centuries, the left hole-shape was modified to resemble an elongated "*f*", with the right hole-shape being its mirror image. Nia, et al. (2015) demonstrates that this modification, when combined with a thicker back, optimally increases the violin's loudness. In the 19th century the neck and fingerboard were lengthened, and the bridge raised to increase the variety of notes and their sound brightness. Many older violins were reworked to reflect this new configuration, including most of the Antonio Stradivari and Giuseppe Guarneri "del Gesù" violins in existence.

Over the years at least three explanations have surfaced to explain the violin similarity phenomenon. The first relates to the master-apprentice model. Sandys and Forster (1864) point out that many apprentices of famous luthiers also became well-known. They not only learned the mechanical techniques necessary to create a violin, but also, they absorbed the master's way of thinking with respect to violin design and subsequent execution. This behavior continued until a breakthrough in design occurred. The new style lasted until the next major innovation. The stability of violin design since the 18th century demonstrates that makers have settled on a "dominant design" as reflected in the work of Utterback (1994).

A second explanation disavows the master-apprentice explanation. Dilworth (2021) argues that the notion that many of the path-breaking luthiers had been students of other well-known luthiers is shaky at best. Instead, he points out that it was not uncommon for luthiers to unabashedly copy the work of others and put an authentic-looking label on the end-product. Dilworth (2021) contends that the similarities of violins are the result of luthiers recognizing technical improvements in the work of others. Thus, they not only incorporated these improvements in their violin designs but also used them as a steppingstone to make other important advances in violin design. Chitwood (2014) is in general agreement with Dilworth's (2021) overall assessment, and he points out that the similarities in design are even more noticeable among the violins made in luthiers in the same multigenerational family.

A third explanation, which contains the first two, involves the intersection of mathematics (especially geometry) and music.[4] Schleske (2004) suggests that the prevailing view is that luthiers who were active in the 1700s and earlier created their violins intuitively. Disagreeing with this position, he (2004, p. 1) opines that these luthiers were extremely capable individuals, and many were "…extremely receptive to methods, ideas and discoveries made in science and the arts of their age, including architecture, the natural sciences, composition and painting." Hence, it is likely that they would ensure their apprentices were familiar with all the tools and procedures needed to construct the most up-to-date violin configuration.

Schleske (2004) specifically mentions mathematics and notes the role of cycloids in determining the shape of a violin's table and back plate arches as well as in the known construction architecture of the time.[5] The latter not only includes the ubiquitous presence of arches in buildings but also in bridges. Playfair (2003), after comparing numerous Cremonese violins to computer-generated cycloid profiles, shows that there is a strong correspondence between cycloid curves and their cross-arch profiles, i.e., those determined by the instrument's width and depth. Schleske (2004) confirms these results and observes that there are systematic differences among the luthiers with respect to their profiles.[6] Playfair (2003) and Schleske (2004) test

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[4] Hutchins (1983) provides a historical development of violins from the late 16th century to the late 20th century. He not only gives an overview of violin making and the implicit and explicit roles of mathematics, but he also relates the instrument to other fields of study such vibration analysis, psychoacoustics, materials, and musical composition and performance.

[5] A cycloid is a locus of points created by a point on a circle's (wheel's) perimeter rolling along a straight line. If the point is located inside the circle on one of the radii (not the center), the result is named a curtate (short) cycloid. If the point is located outside the circle, the locus of points is called a prolate (extended) cycloid. If the circle point rolls repetitively, it is an example of simple harmonic motion. Because of the work of mathematicians such as Nicholas (Kryffs) of Cusa (b. 1401-d. 1464), Galileo Galilei (b. 1564 - d. 1642), Blaise Pascal (b. 1623-d. 1662), Gottfried Wilhelm Leibniz (b. 1646-d. 1716) and Isaac Newton (b. 1643-d. 1727), it is highly likely that early luthiers were aware of cycloids and their characteristics. In addition, Albreht Durer (b. 1471-d. 1528) wrote detailed descriptions of geometry and showed how curves could best be used by artists and craftsmen (Schleske, 2004).

[6] For instance, the actual arches found in Nicolò Amati violins tend to lie within a the-

results support the contention that luthiers experimented with various designs and rejected those that did not provide the desired results but kept the ones that showed improvement. As a result, early luthiers acted as empirical scientists, and today's violin is a "… highly optimized acoustic system…" (Schleske, 2004, p. 1)."

Schleske (2004) also points out that luthiers were confronted by "new" music and, to stay competitive, had to be sure that their violins could meet the demands this music required. For instance, in 1680, Arcangelo Corelli (b. 1653-d. 1713) created the concerto grosso, which increased the violin's playing time and eventually led to it becoming a solo instrument. The violin's importance to music grew even more in 1698 when Giuseppe Torelli (b. 1658-d. 1709) composed his first violin solo concerto. According to Schleske (2004), Antonio Stradivari responded to the composition by distancing his violin designs from those of Nicolò Amati, and his reaction to Torelli's event was to create a new violin model, which initiated what has come to be known as his "golden period.". In the ensuing years, Antonio Vivaldi (b. 1678-d. 1741) wrote over three hundred violin solo concertos and over forty violin duets. The apex of the solo violin literature was achieved around 1720 when Johann Sebastian Bach (b. 1685-d. 1750) completed his Six Sonatas and Partitas for Solo Violin. This interdependence between composers and luthiers led to the widespread adoption of the violin family of instruments.

2.3 Violins as a Cultural and Monetary Investment

Several recent violin studies focus on their viability as an investment. A detailed exploration of the investment characteristics of these violins is supplied by Muñoz (2020). Using auction and private dealer market sales data, she reports that high-quality, collectible violins provide steady annual returns with little volatility. She also finds that Antonio Stradivari and Giuseppe Guarneri "del Gesù" violins

oretical cycloid arch, and those by Giuseppe Guarneri "de Gesù" are more likely outside of the mathematical arch. Antonio Stradivari's arches, however, cluster around the theoretical cycloid arch.

are the preferred choice and, thus, tend to provide the best investment opportunities. Moreover, her study documents the presence of a musician effect, i.e., if the violin is associated with a highly-reputed musician, its average investment return is higher than that for an instrument without a significant player association.[7] Muñoz (2020) reports that the correlation between violin returns and those of financial assets are typically low, suggesting that their returns may be helpful to achieve a desired level of diversification. Her research supports Campbell's (2008) and Graddy and Margolis' (2011) findings that investment grade violins should be considered as an alternative investment but that their presence in an investor's portfolio should be relatively small and consideration should be given to the investor's need for liquidity.

The belief that violins are viable investments is supported by investment funds that focus on stringed instruments. For example, Cremona Capital's (https://cremonacapital.com) business model is purchasing high-end vintage violins and selling them in four to five years. During the holding period, Cremona Capital lends the violins to working musicians, who are responsible for insuring the instruments. These loans not only benefit the music world in general, but also help violins maintain their musical tone. The fund requires that investors not only provide initial capital to join the fund but also pay to have the instrument insured against loss and damage as long as the violin is owned by the fund and not loaned to a musician. In turn, the investors receive 20 percent of the profit when an instrument is sold after paying the fund a one-percent commission.[8]

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[7] Music historians divide the last millennium into six eras: Renaissance (1400-1599), Baroque (1600-1749), Classical (1750-1829), Romantic (1830-1899), 20th Century (1900-1999), and Modern (2000 and after). A list of famous musicians would surely include Antonio Vivaldi (Baroque), Joseph Haydn (Classical), Niccolò Paganini (Romantic), Fritz Kreisler (20th Century) and Krzysztov Penderecki (Modern). Although these eras undoubtedly overlap, they do signify a change in music composition and public taste.
[8] Another business model involving a vintage violin using non-fungible tokens (NFTs) is being considered (Hongji, 2024). A 1709 Stradivari violin valued at $9 million was recently tokenized, i.e., the violin is now not only a physical asset but also a digitalized asset in a blockchain system. The violin's owner is considering using tokens to permit partial ownership.

There are currently three ways to buy and sell physical vintage violins: auction houses, musical instrument dealers, and agreements between private parties. It is not uncommon for auction houses to have close relationships with dealers in order to serve effectively and benefit from both types of clienteles. Auctions (in person, by phone, or web-based) are a common venue to buy or sell violins and are usually run by reputable auction houses. Currently, the major auction houses include Bromptons, Ingels & Hayday (previously a part of Sotheby's), Tarisio, and Vinchy Enchères. The auction house landscape is dynamic, and a similar list in 1999 would have included Bonham's, Christie's, Sotheby's and Phillips (Muñoz, 2020).[9]

Violin auctions are structured as ascending (English) auctions. Buyers compete by sequentially submitting higher bids than the previous bid. The auction ends when the bidding stops, and the final bidder is declared the winner, assuming that the reserve price is met. The purchase price is the final bid price plus a buyer's commission, usually around 20 percent. Since the auction house acts as the seller's agent, the seller also pays a commission. The amount of the bid made by a potential buyer depends on the value of the violin to the buyer and his assessment of its value to the other bidders. Determinants of value include investment potential as well as the prestige that comes from owning the instrument. To assist in assessing a violin's value, an auction house publishes a catalog that describes in some details the violin and features its picture and provenance. Additionally, instrument dealers and independent experts often provide certificates of authenticity for instruments, and instruments often have one or more certificates that identify the maker. During viewing periods prior to the sale, a prospective bidder often is accompanied by an expert to assess the instrument's quality, condition,

[9] In 2007 Sotheby's (Est. 1746) decided to eliminate violins from its auction activities and focus on art and jewelry. Shortly thereafter Christie's (Est. 1776) and Bonham's (Est. 1793) also withdrew from marketing violins. Phillips (Est. 1736) continues to be involved with the instrument but appears to emphasize contemporary art and design. In contrast, Vinchy Enchères (Est. 1983), Tarisio (Est. 1999), Bromptons (Est. 2005) and Ingels & Hayday (Est. 2012) currently focus on musical instruments.

and authenticity. The last step is important because, as Harvey and Shapreau (1997) point out, stolen and sophisticated fake violins may be auctioned, and it is not always possible for the buyer to be compensated for his loss.

A violin dealer typically has an inventory of violins and other string instruments on display in a showroom. The sales staff, which may include the owner(s), are experts in violin history and construction. Nevertheless, a potential buyer may be accompanied by an expert to validate that the violin is what the dealer contends it is. Buying from a showroom also may allow would-be buyers to physically handle the instrument, create a positive relationship with the dealer, and if they are long-term customers, try to exploit their purchasing history to obtain a better deal. Often, dealers will allow players to try out a violin for a period to see if the instrument suits their needs and playing style. From the buyers' perspective, another advantage of using a dealer is that it is unlikely that they would become involved in a bidding war. The dealer's price is usually thought of as a retail price, while the auction price is considered equivalent to the wholesale price. The difference between these two prices, among other possibilities, may be the result of the risk that the buyer takes with respect to the authenticity of the instrument (Harvey and Shapreau, 1997). Dealers may participate in auctions hoping to acquire an investment-grade violin at a price that permits them to resell the instrument at a handsome profit or to spread the risks that are inherent in their violin inventories (Martin Swan Violins, 2021).

In contrast to auction and dealer transactions, the price associated with a private transaction is an amount that is negotiated between an individual buyer and seller. Such a transaction might occur if the buyer and seller were wealthy collectors seeking to modify their holdings. Another transaction may be the seller wanting to donate a valuable violin to a charitable foundation. In both examples, it is likely that the transaction price is different than the auction or dealer prices, and it is usually known only by the two parties involved in the transaction.

Despite the method of sale, Kass (2015) maintains that information travels as swiftly as technology permits. He believes that today's violin buyers and sellers know what the "market will bear" in terms of price. Moreover, a transaction's selling price is quickly known by current and would be investors, even though the general population may not even know that a transaction has occurred. Kass (2015) also points out that historically violins of some luthiers always sell at a higher price than some made by other luthiers. He states that this relative pricing system has become a market custom; consequently, there is an implied quality ranking of luthiers.

3. Market Network Dynamics

Complex behavior is typically defined in terms of influences: in-influence, out-influence, and self-influence. To illustrate, consider the case involving luthiers A and B. In-influence occurs when a transaction by luthier A precedes a transaction by luthier B. Out-influence happens when a transaction by luthier B precedes a transaction by luthier A. Self-influence exists when a transaction of a luthier A (or B) affects a subsequent transaction of luthier A (or B). The three influences are independent of each other so there can be various magnitudes and combinations of influences. The first two influences are often combined, with the combination referred to as either cross-influence or mutual-influence. Cross-influence plus self-influence equals total-influence.

3.1 Modeling Complexity

Graphically, market complexity is modeled as a network of agents with the nodes of the network symbolizing the agent and line segments (edges) between any pair of these nodes, indicating that they are connected. An arrow attached to the end of a line segment signals the time direction of the connection. These intra-market influences can be depicted by a square matrix with its size determined by the number of unique entities being considered. This matrix is typically referred to as an excitation matrix, with, in this case, the columns of the matrix denoting the luthiers providing the influence and the rows identifying those luthiers being influenced. The principal diagonal of the matrix quantifies self-influence.

To construct this matrix, we employ the constant Hawkes (1971, 2018) process, which contains two components: a constant term and the excitation matrix.[10] The constant term contains the mean influence of all the factors not contained in the excitation matrix and is considered as exogenous. In contrast, the contents of the excitation matrix are endogenous, and they serve to model beliefs and other factors that are directly associated with transactions. Only past events are considered so that the entries in the excitation matrix are multiplied by a function that decays with time to reflect the phenomenon that the level of importance of information tends to decrease with the passage of time.[11]

The Hawkes process is similar to a Poisson process augmented by lagged parameters that reflect a decay in importance as the length of the lag increases. The value of the cell entries of this matrix, as demonstrated by Hlaváčková-Schindler, et al. (2024) and Etasami, et al. (2016) in conjunction with Quinn, et al. (2011) can be interpreted using the concept of Granger (1969, 2004) causality: a statistical view that supports the philosophical notion of cause or, in the

[10] The Hawkes process has been used in several research areas. Finance examples include stock market liquidity (Bacry et al., 2015), financial contagion (Aït-Sahalia et al., 2015), financial risk (Soramäki and Cook, 2016), portfolio choice (Aït-Sahalia and Hurd, 2016), insurance (Swish et al., 2021), stock market crashes (Shi et al., 2022), and cryptocurrency (Luo et al., 2022). The approach has also been used in seismology (Ogata, 1998). warfare (Tench et al., 2016), wireless networks (Moore and Davenport, 2016), social media (Rizoiu et al., 2017), infectious diseases (Unwin et al., 2021), and group networks (Fang et al., 2023). Additional details on the nature of the Hawkes process may be found in numerous sources, such as Embrechts et al. (2011) and Laub et al. (2021).

[11] Complexity is typically thought to be a nonlinear concept. This leads Arthur (1999) to argue that a more descriptive term for complexity would be an adaptive nonlinear network. Ladyman et al. (2012), however, state that if trading is involved, the resulting transactions can be expressed by a linear matrix. Thus, the Hawkes excitation matrix represents a special case of complexity.

context of this study, influence. Granger-cause is based on the notion that the occurrence of past events may increase the probability of future events.[12]

3.2 The Hawkes Model

Following current work, we estimate the parameters of the Hawkes process using procedures developed by Linderman and Adams (2014, 2015). These procedures are available on Github (https://github.com/slinderman/pyhawkes).

3.2.1 The Hawkes Model Specification

Let t_i^v denote the time of the ith event of a violin transaction attributed to luthier v. The probability P_v that a transaction occurs in the next dt time units for luthier v is determined by the instantaneous rate $\lambda v(t)$ so that $P_v \approx \lambda v(t) \bullet dt$. This rate is a function of the occurrences of previous events from all violin transactions, i.e.,

$$\lambda_v(t) = \sum_{v'=1}^{n} \sum_{i:t_i^{v'}<t} e_{vv'} g\left(t - t_i^{vt}\right) + \mu_v \quad (1)$$

where $i{:}t_i^{v'} < t$ corresponds to all the events of luthier v' that occurred before time t, and e captures the impact of luthier v' on luthier v. There are $n \times n$ such parameters and these are estimated from the data. The expected number of events of luthier v preceded by an event in luthier v' is e_{vv}'. Because transactions are excitatory interactions, all e_{vv}' are greater than zero. The term $g(t - t_i^{v'})$ describes how the effect from each previous event decays over time. Finally, μ_v is

[12] To illustrate, suppose there are two time series variables (X and Y) and each of them are thought to be predicted by vectors of both their lagged values. Y Granger-causes X if Y's lagged terms are statistically significant when predicting X. X Granger-causes Y if X's lagged terms are statistically significant when predicting Y. If the lagged values X and Y are both statistically significant, the result is a causal feedback loop. If the lagged values of X and Y are both statistically insignificant, there is no evidence of Granger-causality

the baseline rate of events of partition v that is independent from all previous events. It represents all exogenous effects to the violin market such as news of all types and effects emanating from all violins not included in the analysis.

We follow Linderman and Adams (2014) and use the logistic-normal density function to measure decay for three reasons. First, it integrates to one, which endows $e_{vv'}$ with the units of the expected number of events and allows a comparison of the strength of interactions. Second, it naturally models the domain of $g(t - t_i)$, which is bounded below at zero and above by Δt_{max}. Finally, it allows for an efficient Bayesian estimation framework. Compared to standard Hawkes specification, Linderman and Adams (2014) show that their version achieves better results in identifying relationships between events.

Equation (1) can be compactly written using matrices. Let:

$$\lambda(t) = \left(\lambda_1(t), \cdots, \lambda_n(t)\right)^T E = \left[e_{vv'}\right],$$

$$g(t) = \left(\sum_{i:t_i^1 < t} g\left(t - t_i^1\right), \cdots, \sum_{i:t_i^n < t} g\left(t - t_i^n\right) \right)^T,$$

and

$$\mu(t) = \left(\mu_1(t), \cdots, \mu_n(t)\right)^T$$

Equation (1) can be then expressed as:

$$\lambda(t) = E \bullet g(t) + \mu(t) \tag{2}$$

E, the excitation matrix, is the basis of our analysis because it captures the interactions between the luthiers. Specifically, *E*'s columns depict the partitions that trigger the effects, and its rows denote the partitions that are affected. Its principal diagonal represents the self-induced impact (self-influence) on each of the n partitions. The remaining $n(n - 1)$ cells in the excitation matrix measure the impact of the violins in one partition on those in another partition (cross-influence). The contents of *E* can be summarized in terms of density and reflexivity. Density is the sum of all non-zero matrix entries divided by the number of all possible connections. Filimonov and Sornette (2012), among others, point out that there are two types of reflexivity. Cross-reflexivity is the mean impact of the transaction influences of one luthier on all the other luthiers. Self-reflexivity is the mean impact that each luthier has on itself.

Because equation (2) is a static model, the excitation matrix describes the influence structure for the entire period being studied. To investigate the possibility of a time-dependent excitation matrix, we construct a series of overlapping, sequential time windows. Influences contained in the matrixes from these time windows can be thought of as a moving average that may change over time. The granularity of each of the sequential matrixes depends on the number of total observations and the number of observations in each time window.

3.2.2 Estimating the Hawkes Model

We estimate the Hawkes model using the Bayesian method developed by Linderman and Adams (2014) and their Python package "pyhawkes" (https://github.com/slinderman/pyhawkes), which is efficient and reliable in discovering latent network structure. Let $p(\{t_i^v\} \mid \mu, E, g)$ to be the likelihood function of observing the sequences of events $\{t_i^v\}v_{v=1}^n$ given the parameters μ, E, and g, which are well-defined due to the Hawkes model assumptions. Nevertheless, because of the complex nature of the Hawkes likelihood function, there is not a closed-form solution nor is there a straightforward optimization approach.

To address this issue, Linderman and Adams (2014) develop an efficient Bayesian inference e algorithm, which is equivalent to updating its parameters, including the components of the excitation matrix. They (2014) decompose the excitation matrix E into two parts: $E = E' \times W$, where E' is a binary matrix modelling the structure of the network ($E'_{ij} = 1$ if there is an edge between nodes i and j and $E'_{ij} = 0$ otherwise), and W is a non-negative weight matrix modelling the strength of the edges between nodes.[13] Given the likelihood function $p(\{t_i^v\} \mid \mu, E', W, g)$, the Bayesian inference method proceeds as follows.[14] Assuming that $\Pi(\mu)$ is a prior distribution of μ, by Bayes' rule, the posterior distribution of μ is:

$$p\left(\mu \mid \left\{t_i^v\right\}, E', W, g\right) = p\left(\left\{t_i^v\right\} \mid \mu, E', W, g\right) \Pi\left(\mu\right) / p\left(\left\{t_i^v\right\}, E', W, g\right) \quad (3)$$

We sample E', W, g, and μ values from their posteriors to construct their estimates. This process is repeated multiple times, and the sample average is used as the estimate for each parameter.

4. Data and Descriptive Information

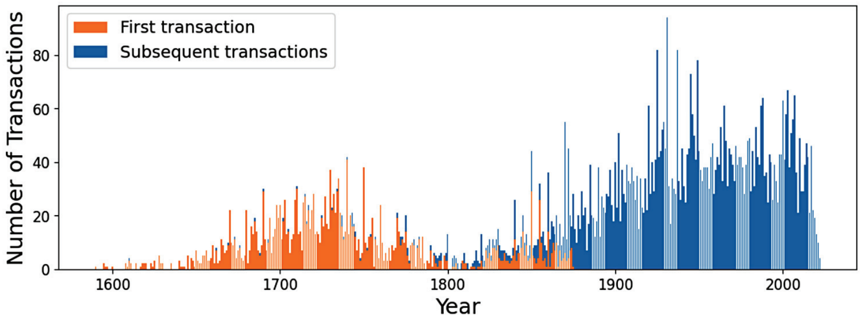
The primary source of the data used in this study is the historical musical instrument archive developed and maintained by Tarisio (https://tarisio.com/cozio-archiv/browze-the-archiv/makers/). Additional data are obtained from Graddy and Margolis (2011) (http://people.brandeis.edu/~kgraddy/data.html). The Tarisio data includes the year of sale for all transactions, but prices are not always available, especially for the early years. Thus, although we use the recent price information to identify top luthiers, the fundamental unit of our analysis is a violin transaction.

Our focus is on violin transaction data pertaining to the top 30 luthiers. We construct this group using the highest nominal auction price (measured in U.S. dollars) ever achieved for each luthier as of June 2022. In total there are 8,083 observations and 2,209 violins. The transactions by year from 1590 to June 2022 are shown in Figure 1. The initial transaction for each violin is plotted in orange. For the most part these transactions are those associated with the luthier delivering the violin to the original client. All subsequent transactions

[13] The advantage of this separation is that random graph models can be used to describe the network structure and impose separate beliefs about the strength and the structure of the network as Bayesian prior probabilities.
[14] This is an example of Gibbs sampling (a procedure using a Markov Chain - Monte Carlo algorithm) with modifications (Linderman and Adams, 2015).

are plotted in blue. From 1590 to the middle 1800s, initial transactions far outnumber secondary transactions and are likely the result of privacy and bookkeeping issues. Subsequently this dominance is reversed.

FIGURE 1
Number of Violin Transactions by Year (1590-2022)



Since we are interested in the transaction dynamics of each luthier, the transactions sample must exhibit the existence of partitions.[15] Partitions occur when there is some commonality among the transactions. To examine the presence of these structures in the violin network, we use the modularity (MOD) statistic, which we calculate using Traag et al. (2019)'s Github Python package (https://github.com/vtraag/leidenalg). MOD uses the excitation matrix to locate communities of nodes that show strong internal connections with each other. A MOD value of zero indicates a random network, and positive values indicate the presence of communities. The yearly entries are calculated using a 20-year moving sample with a one-year step ahead beginning in 1590. The MOD values differ in strength, but all are greater than zero, indicating the presence of partitions.

[15] The number of observations per violin is 3.7, which does not allow rigorous analysis of the interactions between violins. In contrast, the average number of luthier interactions is 269.4.

The high to low ranking in dollar terms of the top 30 luthiers is given in Table 1. Included in this table are the transaction amount, the year the transaction was made, the city where the violin was manufactured, and the auction house involved in the transaction. The top sale price attributed to a specific luthier range from $327 thousand to $15.8 million. Antonio Stradivari ($15.8 million) tops the list and is distantly followed by Giuseppe Guarneri "del Gesù" ($3.6 million) and Giovanni Guadagnini ($2.1 million). Carlo Bergonzi I ($1.0 million) is fourth and followed by a smooth price decline until Nicholas Lupot ($327 thousand) who is 30th. All these sales are from auctions that occurred between 2005 and 2022. The 30 violins were made in seven different cities located in present-day Italy with the remainder being in present-day France and present-day Austria. Five auction houses handled the sales of these violins. Sotheby's oversaw the auctions of three violins. Brompton's and Ingles & Hayday successfully auctioned nine violins, respectively. Tarisio sold eight, including the highest and third-highest priced instruments. Arguttes engineered the second-highest recorded auction price.

A summary of the transactions by luthier is given in Table 2. The transactions begin in 1590 and end in June 2022. Antonio Stradivari is the most prolific with 517 violins associated with 3,177 transactions. Using the number of transactions as the measurement metric, the second and third most prolific are Giuseppe Guarneri "del Gesù" and Giovanni Guadagnini, with 834 and 727, respectively. If the number of known violins is used to judge productivity, the second and third positions are held by Giovanni Guadagnini (241) and Jean-Baptiste Vuillaume (235). The manufacturing time spans of the three most prolific using transactions overlap with one another and range from 1666 to 1786. In contrast, Vuillaume was a luthier from 1823 to 1875. Many of the top 30 luthiers are likely to have been more productive than indicated in our data. For example, the bottom three, based on the number of violins, are Francesco Goffriller, Pietro Giovanni Mantegazza, and Vincenzo Rugeri, with the numbers being 13 and below. The transaction numbers give a similar result. Gof-

TABLE 1
Descriptive Data for the Highest Sales Price Ever: Top 30 Luthiers

Luthier	Rank	U.S. $ Amount	Year	City	Auction House
Stradivari, Antonio	1	15,821,285	2011	Cremona	Tarisio
Guarneri del Gesù, Giuseppe	2	3,637,892	2022	Cremona	Aguttes
Guadagnini, Giovanni Battista	3	2,106,933	2018	Turin	Tarisio
Bergonzi I, Carlo	4	1,001,384	2005	Cremona	Sotheby's
Amati II, Girolamo	5	917,453	2018	Cremona	Ingles & Hayday
Montagnana, Domenico	6	903,934	2010	Venice	Brompton's
Guarneri (of Mantua), Pietro	7	892,685	2020	Mantua	Brompton's
Storioni, Lorenzo	8	664,821	2017	Cremona	Brompton's
Guarneri (of Venice), Pietro	9	662,852	2017	Venice	Ingles & Hayday
Amati, Nicolò	10	654,821	2013	Cremona	Ingles & Hayday
Guarneri "filius Andreae", Giuseppe	11	654,237	2019	Cremona	Ingles & Hayday
Balestrieri, Tomasso	12	621,575	2015	Mantua	Ingles & Hayday
Vuillaume, Jean-Baptiste	13	542,369	2021	Paris	Ingles & Hayday
Guarneri, Andrea	14	542,297	2014	Cremona	Ingles & Hayday
Pressenda, Giovanni Francesco	15	527,020	2019	Turin	Brompton's
Rogeri, Giovanni Battista	16	519,004	2011	Brescia	Sotheby's
Rugeri, Vincenzo	17	502,320	2011	Cremona	Brompton's
Amati, Antonio & Girolama	18	447,200	2011	Cremona	Tarisio
Rocca, Giuseppe Antonio	19	412,624	2018	Turin	Brompton's
Serafin, Santo	20	403,990	2014	Venice	Ingles & Hayday
Goffriller, Francesco	21	379,794	2017	Venice	Brompton's
Rugeri, Francesco	22	375,153	2015	Cremona	Tarisio
Stradivari, Omobono Felice	23	370,000	2005	Cremona	Tarisio
Tononi, Carlo Annibale	24	362,586	2010	Venice	Brompton's
Gagliano, Nicola	25	355,000	2021	Naples	Tarisio
Camilli, Camillo	26	349,000	2021	Mantua	Tarisio
Gagliano, Alessandro	27	344,229	2014	Naples	Tarisio
Mantegazza, Pietro Giovanni	28	334,322	2010	Milan	Brompton's
Stainer, Jacob	29	331,768	2011	Absam	Sotheby's
Lupot, Nicolas	30	327,119	2019	Paris	Ingles & Hayday

Sources: https://tarisio.com/cozio-archiv/browze-the-archiv/makers/
http://people.brandeis.edu/~kgraddy/data.html

TABLE 2
Summary of Vintage Violin Transaction Data: 30 Luthiers

Luthier	Years Made	# of years	# Made	# Made per year	Transactions		
					Total	Per Violin	Max. per Violin
Stradivari, A.	1666-1740	72	517	7.93	3177	6.15	20
Guarneri "del Gesù", G.	1719-1744	26	136	2.96	834	6.13	14
Guadagnini, G.B.	1740-1786	47	241	5.13	727	3.02	9
Bergonzi I, C.	1715-1740	26	37	1.42	201	5.43	11
Amati II, G.	1671-1708	38	17	0.45	38	2.24	6
Montagnana, D.	1710-1742	33	32	0.97	95	2.97	8
Guarneri, P. (Mantua)	1676-1717	42	40	0.95	119	2.98	9
Storioni, L.	1768-1796	29	41	1.41	100	2.45	7
Guarneri, P. (Venice)	1720-1757	38	34	0.89	97	2.87	8
Amati, N.	1626-1684	57	86	1.51	242	2.83	7
Guarneri, G (filius A.)	1689-1730	42	68	1.62	189	2.78	7
Balestrieri, T.	1750-1792	43	41	0.95	92	2.24	4
Vuillaume, J.-B.	1823-1875	53	235	4.43	494	2.10	8
Guarneri, A.	1640-1696	57	64	1.12	156	2.44	7
Pressenda, G.F.	1819-1843	25	73	2.93	171	2.34	6
Rogeri, G.B.	1671-1712	42	46	1.10	118	2.57	6
Rugeri, V.	1663-1718	56	13	0.23	40	3.08	7
Amati, A. & G.	1590-1635	46	34	0.97	95	2.79	8
Rocca, G.A.	1831-1865	36	45	1.29	112	2.49	7
Serafin, S.	1720-1755	36	41	1.14	101	2.46	11
Goffriller, F.	1719-1739	21	12	0.57	22	1.83	5
Rugeri, F.	1650-1700	49	48	0.98	116	2.42	9
Stradivari, O.F.	1689-1740	50	20	0.38	51	2.55	4
Tononi, C.A.	1700-1737	38	29	0.53	67	2.31	5
Gagliano, N.	1727-1783	57	90	1.58	192	2.13	6
Camilli, C.	1730-1759	30	37	1.63	82	2.22	7
Gagliano, A.	1700-1734	35	24	0.69	62	2.58	9
Mantegazza, P.G.	1760- 1790	31	12	0.39	24	2.00	4
Stainer, J.	1645-1680	36	56	1.56	167	2.98	7
Lupot, N.	1755-1824	70	40	0.57	102	2.55	8
Total			**2,209**		**8,083**		
Minimum	1590	21	12	0.23	22	1.83	4
Median		40	41	1.11	107	2.55	7
Mean		42	74	1.61	296	2.86	7.6
Maximum	1875	72	537	7.93	3177	6.15	20

Sources: https://tarisio.com/cozio-archiv/browze-the-makers/
https://people.brandeis.edu/~kgraddy/data.html

friller and Mantegazza retain their bottom ranking with 22 and 24, respectively. Vincent Rugeri who has 40 is replaced by Girolamo Amati II with 38 transactions.

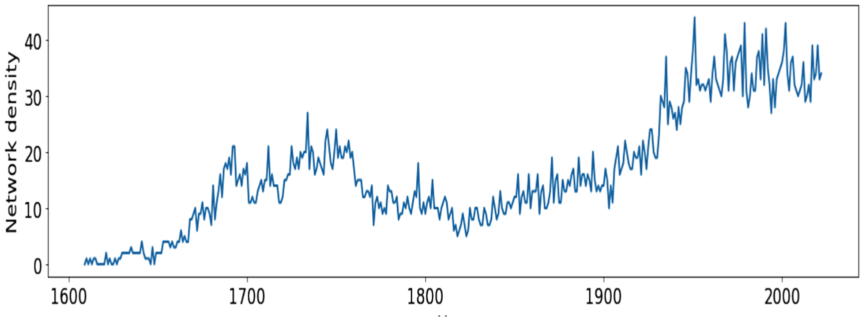
5. Dynamic Complex Behavior Statistical Findings

Before examining the excitation matrix and discussing its dynamic nature as defined by its self- and cross-excitation properties, two issues must be addressed. First, the Hawkes model requires that no event occurs at the same time. Our transaction data, however, are yearly and there are numerous instances of more than one transaction in any given year. To resolve this issue, we randomly add a very small number to each transaction year. This permits within-the-year observations to be sequenced without impacting those in the years adjacent to it. Second, recall that we use a rolling sample to gain insights into how the influences change over time. Thus, the samples are not necessarily the same size because their definition depends on equal time segments (years) rather than an equal number of transactions.

5.1 Network Density (1590-2022)

As mentioned in Section 3.2.1, network density is the percentage of total cells of an excitation matrix that are nonzero. Thus, it is a basic measure of how transactions influence each other, with a cell measure of zero indicating no influence at all. Because our measure is a rolling 20-year sample with a one-year step ahead, we can examine how this measure changes on a yearly basis time. The dynamic network density percentages for 30 luthiers are given in Figure 2. As shown in this figure, there is an upward trend to density. This trend reflects not only an increase in transactions over time but also indicates that trading has become more diverse.

FIGURE 2
Network Density by Year (1590-2022)

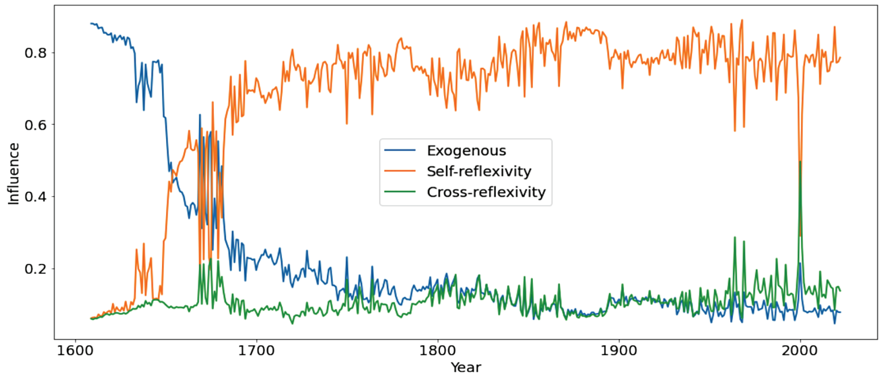


5.2 Influences (1590-2022)

The relative importance of exogenous, self-reflexivity, and cross-reflexivity is depicted in Figure 3. As discussed in Section 3, exogenous influences are those factors that may influence violin transactions but do not directly relate to the violins themselves. In contrast, self-influence is the impact that a transaction of a luthier has on himself, while cross-influence is the impact that a transaction of a luthier has on another luthier, i.e., the sum of in-influence and out-influence. The graph's y-axis shows the percentage of total influence associated with each of the three types of influence for each year of the sample. The sum of the three influences in each year equals 100 percent.

A review of this figure shows that exogenous influences dominated the other two influences until the middle 1600s. At that point self-reflexivity influence, with a few notable exceptions, became the dominant influence. The exceptions are highlighted by the upward spikes and downward spikes. In the late 1600s, exogenous and self-reflexivity influences often switched positions as the dominant effect. In contrast, circa 2000 there are times when the exogenous influence is around 10 percent of the total influence, In contrast, cross- and self-reflexivity influences account equally for the remaining 90 percent.

FIGURE 3
Influences: Exogenous, Self-Reflexivity,
and Cross-Reflexivity by Year (1590-2022)



5.3 Influences: A Two-Sample Comparison (1590-1875 and 1876-2022)

Sections 5.1 and 5.2 focused on groups of luthiers. In this section we examine and compare the influence characteristics of the individual luthiers. To accomplish this task, we divide the sample into two time periods: 1590-1875 and 1876-2022. The sample split recognizes that the last violin made in the data is 1875, ensuring that all transactions after this date do not involve the luthier himself. In Appendix A we provide the effect and rank of the in-influence, out-influence, and self-influence for each of the 30 luthiers in Tables A1 and A2. The rank correlation coefficient for total-influence between the two periods is 0.7061 ($p < 0.0001$, two-tail test), indicating moderately strong positive correlation between the influence rankings between the two periods.

As shown in Table 3A self-influence is the dominant factor, although the relative contribution to total influence is not the same for the two time periods. The proportion of in-influence decreased from the first to the second period by 7.34 percentage points while the proportions of out-influence and self-influence increased by 0.21 and 7.13 percentage points, respectively. A review of these data indicates a wide disparity of influence among the luthiers, indicating that only a small number of them are what might be called "super- influencers." We define this selected group to be the top 20 percent in the total-influence category. As reported in Table 3B, this group accounts for 59.01 percent of the total influence in the 1590-1875 period and increases to 81.45 percent in the 1876-2022 period. In both periods, as depicted in Table 3C, the super-influencer's self-influence accounts for slightly more than 90 percent of the total influence in both sub-periods.

Table 3
Influence Allocation

	In-influence (%)	Out-influence (%)	Self-influence (%)	Total-influence (%)
All 30 Luthiers: Influence Percentages				
1590-1875	18.11	10.55	71.34	100.00
1876-2022	10.77	10.76	78.47	100.00
Change	-7.34	0.21	7.13	0.00
Top 6 Luthiers: Influence Percentages Relative to all 30 Luthiers				
1590-1875	2.88	2.70	53.43	59.01
1876-2022	4.30	3.73	73.42	81.42
Change	1.42	1.03	19.99	22.41
Top 6 Luthiers: Influence Percentages				
1590-1875	4.87	4.56	90.57	100.00
1876-2022	5.29	4.55	90.16	100.00
Change	0.42	-0.01	-0.41	0.00

Note: Cross-influence (a.k.a. mutual influence) is the sum of in-influence and out-influence.

Source: Derived from Tables A1 and A2.

The names of the six super-influencers and their corresponding transaction information from Tables A1 and A2 are provided in Table 4. For each of the two periods, there are six names, and four of the luthiers are listed in both periods, resulting in a total of eight luthiers being in the top 20 percent at least once. The ranks of the eight luthiers for 1590-1875 and 1876-2022 are in parentheses. As shown in this table, only Antonio Stradivari and Jean-Baptiste Vuillaume did not change ranks, being first and third, respectively. Carlo

Bergonzi I and Giuseppe Guarneri (filius Andreae) moved into the top six. Giuseppe Guarneri "del Gesù" moved to second, while Giovanni Guadagnini dropped to fourth place. Nicolò Amati dropped out of the top six but remained a close seventh. Giovanni Francesco Pressenda, however, dropped from sixth to fourteenth. The reason for both exiting from the top 20 percent is a substantial loss in self-influence.

TABLE 4
Luthier Super-Influencers

Luthier	In-influence Effect (Rank)	Out-influence Effect (Rank)	Self-influence Effect (Rank)	Total-influence Effect (Rank)	Sample Dates and Influence Changes
	0.971 (01)	1.006 (01)	31.434 (01)	33.411 (01)	1590-1875
Stradivari, A.	1.799 (01)	1.224 (01)	80.174 (01)	83.197 (01)	1876-2022
	0.828 (02)	0.218 (04)	48.740 (01)	49.786 (01)	Change
	0.543 (06)	0.581 (03)	6.313 (05)	7.437 (05)	1590-1875
Guarneri, "del Gesù", G.	1.386 (02)	1.147 (02)	8.460 (02)	10.993 (02)	1876-2022
	0.843 (01)	0.566 (01)	2.147 (02)	3.566 (02)	Change
	0.531 (07)	0.522 (06)	8.095 (03)	9.148 (03)	1590-1875
Vuillaume, J.-B.	0.714 (04)	0.787 (04)	9.631 (04)	10.681 (03)	1876-2022
	0.183 (04)	0.265 (03)	1.536 (03)	1.984 (03)	Change
	0.622 (03)	0.520 (07)	13.578 (02)	14.720 (02)	1590-1875
Guadaginni, G.	1.333 (03)	0.964 (03)	5.778 (03)	8.055 (04)	1876-2022
	0.711 (03)	0.444 (02)	-7.790 (30)	-6.655 (30)	Change
	0.395 (24)	0.452 (14)	0.695 (18)	1.542 (19)	1590-1875
Bergonzi I, C.	0.480 (05)	0.595 (06)	0.518 (05)	1.598 (05)	1876-2022
	0.085 (05)	0.028 (11)	-0.177 (15)	0.290 (16)	Change
	0.441 (11)	0.442 (16)	1.409 (13)	2.292 (14)	1590-1875
Guarneri, G.	0.456 (07)	0.594 (07)	0.479 (06)	1.529 (06)	1876-2022
	0.015 (09)	0.152 (05)	-0.930 (04)	-0.763 (20)	Change
	0.611 (04)	0.497 (10)	7.019 (04)	8.127 (04)	1590-1875
Amati, N.	0.478 (06)	0.623 (05)	0.407 (07)	1.508 (07)	1876-2022
	-0.133 (28)	0.126 (06)	-6.612 (29)	-6.619 (29)	Change
	0.562 (05)	0.474 (11)	4.921 (06)	6.993 (06)	1590-1875
Pressenda, G.F.	0.399 (15)	0.489 (12)	0.266 (29)	1.154 (14)	1876-2022
	-0.163 (29)	0.015 (13)	-4.655 (28)	-5.839 (28)	Change

Note: Cross-influence (a.k.a. mutual influence) is the sum of in-influence and out-influence.

Source: Derived from Tables A1 and A2.

Do the network results compare to the beliefs of today's businesses that sell and repair vintage violins? To address this question, we use the ranking data provided by Famous Violin Makers List, which is gathered by Google from a variety of sources from the World Wide Web. This list is compiled from luthier ranking data created by violin sales and repair shops, and, as of August 2024, it contains 26 vintage violin luthiers.[16] As shown in Table 5, the corresponding ranks for the current study are one, seven, and two, respectively. Table 2 and the Famous Violin Makers List have 15 luthiers in common. The Spearman rank correlation between these two groups is 0.629 (p = 0.014, two-tailed test), which indicates a reasonably strong positive statistical relationship despite the relatively small sample size. This rank correlation is stronger than the one between the network rank and the highest sales price (0.557, p = 0.036) and gives some support for Kass' (2015) belief that violin prices may be implied luthier quality measure. The rank correlation between FVML and the highest sales price, although positive, is statistically insignificant (0.291, p = 0.292).

TABLE 5
Comparative Rankings for Network, Famous Violin Makers, and Highest Sales Price

Luthier	Network Rank	FVML Rank	HSP Rank	Luthier	Network Rank	FVML Rank	HSP Rank
Stradivari, A.	01	01	01	Guarneri, P. (Venice)	11	13	09
Guarneri "del Gesù2, G.	02	03	02	Stainer, J.	13	17	29
Vuillaume, J.-B.	03	20	13	Guarneri, P. (Mantua)	15	25	07
Guadagnini, G.B.	04	10	03	Rugeri, F.	16	06	22
Bergonzi I, C.	05	07	04	Montagnana, D.	18	23	06
Amati, N.	07	02	10	Storioni, L.	20	16	09
Guarneri, A.	09	05	14	Lupot, N.	24	24	30
Rogeri, G.B.	10	21	16				

Note: The Famous Violin Makers List (FVML) is constructed by Google and, like the network rankings (Table A2) and highest sales price (HSP, Table 1) rankings, the list can change as new information is discovered, digested, and acted upon by violin buyers and sellers.

[16] Examples of these sources include Amorim Fine Violins (Italy), The Long Island Violin Shop (U.S.), and European Violins (Belgium).

6. Concluding Remarks

Vintage violins (those that are roughly at least 150 years old) are routinely bought and sold through auctions, violin dealers, and private arrangements. These transactions can be depicted by a directed graph based on the date (in our case, the year) of the sale. To quantify this phenomenon, we use Hawkes (1971, 2018) excitation matrix and interpret its contents using the concepts of Granger's (1969, 2004) causality. The unravelling of this network of influences permits individual investors to understand better the dynamics of the violin market, which is essential if the market is fragmented in such a way that the information does not flow freely to all concerned parties (Gomber, et al., 2017). Empirically, we find strong evidence that luthier transactions affect the future transactions of other luthiers (as measured by cross-influence), but by far the strongest effect is self-influence. Antonio Stradivari, Giuseppe Guarneri "del Gesù," Jean-Baptist Vuillaume, and Giovanni Guadagnini continue to dominate the market, although Guadagnini has lost self-influence over time.

The results of our study provide the framework for future research in at least three ways. First, they explicitly and quantitively add the notion of influence networks to the current factors that are used by the music instrument industry to assess the level of fame associated with an individual luthier. Currently the factors include several violin-specific items such as its provenance, the physical condition of the instrument, and the names of the well-known musicians who have publicly played the violin (i.e., the musician effect). Second, much of the published prior empirical research involving violins as alternative investments uses tools related to modern portfolio theory, which is subject to some questionable assumptions such as asset liquidity, investor rationality, efficient markets, and Gaussian distributions. Our approach provides a different perspective. It supports Aït-Sahalia and Hurd's (2016) effort to provide an interesting and useful start to approaching portfolio construction from a complex network point of view. It paves the way for similar research in other markets such as those specializing in fine art, sports memorabilia, rare gold and silver coins, antique and classic automobiles, and real estate to name several possibilities.

Viewing these and similar markets from a complex perspective formalizes a way to understand and think about the market interactions of participants. This knowledge not only will help investors to decide if violins should be represented in their portfolio as an alternative investment class, but also how all the standard investment classes relate to one another. This decision, however, is not solely based on economic benefits. Investing in violins not only helps preserve cultural assets but also often permits them to be used by well-known practicing musicians, thereby allowing music fans and other interested parties the benefit of enjoying live music from an instrument made in Europe's distant past.

Appendix A

Tables A1 and A2 provide the relevant individual luthier in-, out-, and self-influences for 1590-1875 and 1876-2022, respectively.

TABLE A1
Luthier In-, Out-, and Self-Influence: 1590-1875

Luthier	In-influence Effect (Rank)	Out-influence Effect (Rank)	Self-influence Effect (Rank)	Total-influence Effect (Rank)
Amati, A. & G.	0.375 (30)	0.342 (30)	0.902 (11)	1.619 (17)
Amati, N.	0.611 (04)	0.497 (10)	7.019 (04)	8.127 (04)
Amati II, G.	0.399 (23)	0.395 (25)	0.442 (22)	1.236 (24)
Balestrier, T.	0.403 (20)	0.445 (15)	2.109 (09)	2.597 (11)
Bergonzi I, C.	0.395 (24)	0.452 (14)	0.695 (18)	1.542 (18)
Camilli, C.	0.441 (11)	0.439 (17)	0.986 (16)	1.866 (16)
Gagliano, A.	0.403 (20)	0.386 (26)	0.421 (25)	1.210 (27)
Gagliano, N.	0.422 (14)	0.547 (02)	1.374 (14)	2.343 (13)
Goffriller, F.	0.412 (16)	0.366 (29)	0.423 (23)	1.201 (28)
Guadagnini, G.	0.622 (03)	0.520 (07)	13.578 (02)	14.720 (02)
Guarneri, A.	0.508 (08)	0.528 (05)	1.547 (12)	2.583 (12)
Guarneri "del Gesù", G.	0.543 (06)	0.581 (03)	6.313 (05)	7.437 (05)
Guarneri, G. (filius A.)	0.441 (11)	0.442 (16)	1.409 (13)	2.292 (14)
Guarneri, P. (Mantua)	0.401 (22)	0.454 (13)	0.471 (21)	1.326 (21)
Guarneri, P. (Venice)	0.391 (26)	0.436 (19)	0.412 (29)	1.239 (23)
Lupot, N.	0.458 (10)	0.500 (09)	1.712 (11)	2.670 (10)
Mantegazza, P.G.	0.390 (28)	0.377 (28)	0.418 (26)	1.185 (30)
Montagnana, D.	0.386 (29)	0.416 (21)	0.402 (30)	1.214 (25)
Pressenda, G.F.	0.562 (05)	0.474 (11)	4.921 (06)	5.957 (06)
Rocca, G.A.	0.412 (16)	0.432 (20)	1.077 (15)	1.921 (15)
Rogeri, G.B.	0.440 (13)	0.401 (24)	2.079 (10)	2.920 (09)
Rugeri, F.	0.409 (18)	0.546 (04)	0.525 (19)	1.480 (19)
Rugeri, V.	0.393 (25)	0.383 (27)	0.423 (23)	1.199 (29)
Serafin, S.	0.420 (15)	0.437 (18)	0.418 (26)	1.275 (22)
Stainer, J.	0.673 (02)	0.517 (08)	2.367 (08)	3.557 (07)
Storioni, L.	0.478 (09)	0.457 (12)	2.386 (07)	3.321 (08)
Stradivari, A.	0.971 (01)	1.006 (01)	31.434 (01)	33.411 (01)
Stradivari, O.F.	0.406 (19)	0.403 (23)	0.521 (20)	1.330 (20)
Tononi, C.A.	0.391 (26)	0.405 (22)	0.417 (28)	1.213 (26)
Vuillaume, J.-B.	0.531 (07)	0.522 (06)	8.095 (03)	9.148 (03)

Note: Cross-influence (a.k.a. mutual influence) is the sum of in-influence and out-influence.

Table A2
Luthier In-, Out-, and Self-Influence: 1876-2022

Luthier	In-influence Effect (Rank)	Out-influence Effect (Rank)	Self-influence Effect (Rank)	Total-influence Effect (Rank)
Amati, A. & G.	0.364 (24)	0.429 (19)	0.309 (22)	1.102 (19)
Amati, N.	0.478 (06)	0.623 (05)	0.407 (07)	1.508 (07)
Amati II, G.	0.385 (17)	0.286 (28)	0.361 (11)	1.032 (27)
Balestrier, T.	0.387 (16)	0.362 (23)	0.333 (16)	1.082 (21)
Bergonzi I, C.	0.480 (05)	0.595 (06)	0.518 (05)	1.593 (05)
Camilli, C.	0.376 (18)	0.360 (24)	0.318 (19)	1.054 (25)
Gagliano, A.	0.360 (26)	0.382 (22)	0.337 (15)	1.079 (22)
Gagliano, N.	0.411 (11)	0.510 (10)	0.286 (26)	1.207 (11)
Goffriller, F.	0.401 (13)	0.240 (30)	0.352 (12)	0.933 (30)
Guadagnini, G.	1.333 (03)	0.964 (03)	5.788 (03)	8.085 (04)
Guarneri, A.	0.400 (14)	0.531(09)	0.380 (09)	1.311 (09)
Guarneri "del Gesù", G.	1.386 (02)	1.147 (02)	8.460 (02)	10.933 (02)
Guarneri, G. (filius A.)	0.456 (07)	0.594 (07)	0.479 (06)	1.529 (06)
Guarneri, P. (Mantua)	0.376 (18)	0.508 (11)	0.267 (28)	1.151 (15)
Guarneri, P. (Venice)	0.443 (08)	0.446 (16)	0.318 (19)	1.207 (11)
Lupot, N.	0.353(29)	0.436 (18)	0.284 (27)	1.073 (24)
Mantegazza, P.G.	0.425 (09)	0.250 (29)	0.350 (14)	1.025 (28)
Montagnana, D.	0.414 (10)	0.421 (21)	0.287 (25)	1.112 (18)
Pressenda, G.F.	0.399 (15)	0.489 (12)	0.266 (29)	1.154 (14)
Rocca, G.A.	0.357 (28)	0.465 (15)	0.401 (08)	1.323 (08)
Rogeri, G.B.	0.359 (27)	0.483 (13)	0.366 (10)	1.208 (10)
Rugeri, F.	0.372 (21)	0.472 (14)	0.304 (23)	1.147 (16)
Rugeri, V.	0.373 (20)	0.353 (26)	0.348 (13)	1.074 (23)
Serafin, S.	0.361 (25)	0.441 (17)	0.311 (21)	1.113 (17)
Stainer, J.	0.408 (12)	0.536 (08)	0.253 (30)	1.197 (13)
Storioni, L.	0.366 (23)	0.429 (19)	0.290 (24)	1.085 (20)
Stradivari, A.	1.799 (01)	1.224 (01)	80.174 (01)	83.197 (01)
Stradivari, O.F.	0.367 (22)	0.357 (25)	0.327 (17)	1.051 (26)
Tononi, C.A.	0.353 (29)	0.340 (27)	0.326 (18)	1.019 (29)
Vuillaume, J.-B.	0.714 (04)	0.787 (04)	9.361 (04)	10.861 (03)

Note: Cross-influence (a.k.a. mutual influence) is the sum of in-influence and out-influence.

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