Development and Crisis in the Late Middle Ages: the Role of Trade

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The principal aim of this paper is to highlight the influence of trade, and above all of its decline, during the fourteenth-century crisis. To help us we will use a mathematical model, based on a production function that includes a specialisation factor correlated with the monetisation level of the economy, which we can consider a good indication of the specialisation of an economic system[1].

1. Late medieval Europe between growth and crisis

The origin of the fourteenth century crisis and the duration and the nature of its effects have been the subject of great debate. Two schools of thought concerning the origin of the crisis have emerged. On the one hand it has been stated that, due to a Malthusian-type population explosion (Postan, 1950, Duby, 1962, Bridbury, 1973[2], Abel, 1978 [1966]), in the fourteenth century Europe had reached such a high population level that farmers were forced to farm infertile land, and that this was the cause of the gradual decline in the population, which, together with climatic factors (Duby, 1992 [1973], pp. 7-14), led to increasingly serious epidemics; these, however, were supposedly responsible for redressing the balance between resources and population. However Michael Morineau (1998), writing about these epidemics, has objected that "the decline in the Black Death's virulence, as implied by the neo-Malthusian theory applied to the fourteenth century, does not ... withstand examination", because "people did not need to lived in crowded

[1] I would like to thank professor Guglielmo Forges Davanzati of the University of Lecce for the useful discussion I had with him about subjects concerning the theme of this paper.

[2] In his later work, *Before the Black Death* (1977), however, Bridbury suggested that England was not overpopulated in the period before the Black Death.

conditions to fall victim". Lopez (1966 [1962], pp. 35-36) has argued instead that plagues were connected with "climatic pulsations"; this view is now shared by many scholars, such as Livi Bacci (2002 [1998], pp. 61-68).

On the other hand, other scholars, such as Robinson (1959), Miskimin (1964) and above all (for the fifteenth century) Nightingale (1997), have explained the recession using a monetary explanation, while Wolff (1975) already suggested that "...the two explanations, the demographic and the monetary, need to be combined".

We know that the process of expansion in the European economy was driven above all by a series of political, cultural and economic factors, the most important of which was the gradual disappearance of invasions from the tenth and the eleventh centuries onwards[3], with the long period of relative peace that followed in Europe (Duby, 1992 [1973], pp. 143-154). This is pointed out by G. Duby, who, however, does not neglect climatic factors. This period of stability, therefore, came after a time of prolonged and repeated invasions and incursions, which had caused large sectors of the rural population to settle in towns for defence purposes, thus triggering off a process of military and economic development which would in the long run enable Christian Europe to mount a counter-offensive against the raiders (Duby, 1992 [1973], pp. 148-149). One of the decisive factors in the economic take-off was the introduction of the heavy plough. This was made possible when oxen were replaced by horses, which became more available for use in agriculture when the invasions ceased. Together with the cultural changes due to the more widespread penetration of Christianity, which did away with the pagan custom of burying the dead person with his work tools and money, thus preventing useless hoarding (Duby, 1992 [1973], pp. 67-70 and p. 160), it was the adoption of the heavy plough and the introduction of other more advanced methods of cultivation that allowed a surplus to accumulate. Many other innovations appeared or, more simply, came into general use only in that period (Cipolla, 1985, pp. 185-204, Ashtor, 1989). For example, the water-wheel, which had already appeared in the first century B.C. at the eastern

[3] It should be added, by the way, that the climate was already favourable at the time of these invasions: in fact, in that period (tenth century) the colonisation of Greenland took place (called Green Land, precisely, by Erik Rauda) at the hands of the Vikings.

frontiers of the Roman Empire (Lopez, 1966 [1962], p. 28), was used until the tenth century almost exclusively for the grinding of grain, and it was not until the twelfth century that it began to be used for numerous other products (Cipolla, 1985, pp. 188-189)[4].

All these innovations and circumstances enabled the population to grow and another part of the population to settle in towns. The opening of this frontier (the town walls) (Pirenne, 1969 [1936], p. 163, Cipolla, 1985, pp. 165-166) was accompanied by others: expansion towards the west (the Iberian peninsula), the north-east (Teutonic knights) and the south-east (Crusades). Christian expansion in the Iberian peninsula, in particular, determined a steady flow of gold and silver, plundered during the frequent raids on Islamic lands. This led to the progressive transition to a monetary economy, firstly in the areas nearest to the front, then in those further away (Duby, 1992 [1973], pp. 179-182). Naturally the Christian expansion in the Iberian peninsula and in other territories was neither the sole nor the principal cause of the monetisation of the European economy, whose driving force was the discovery of new silver deposits in Germany which could be mined (Spufford, 1988, pp. 74-75, Kohn, 1999). Medieval manufacturers, moreover, entrusted their fortunes more and more frequently to the market (Britnell, 1993 and Britnell and Campbell, 1995).

All this brought back the use of money (Rutenburg, 1983, pp. 113-133) and enabled trade to expand to a degree not seen since the time of the Roman Empire. This led, especially in the twelfth century, to an increasing specialisation of production in the various European regions and to their increasing economic interdependence.

Last but not least, among the causes of the "Commercial Revolution", we must not forget the growth of shipping. Pirenne (1922, 1923, 1927, 2000 [1937]) recognised that the Mediterranean sea and its trade was the most decisive factor influencing the entire economy of medieval Europe. Without enlarging on his theory about the exact historic moment in the early Middle Ages when European trade in the Mediterranean declined, we have to acknowledge the fact, as Fossier (1985 [1982], p. 280) stressed,

[4] A wide and detailed description of the history of the vertical water-wheel is provided by Reynolds (1983) and further information is found in Munro (2003).

that the reconquest of the Tyrrhenian Sea by Pisa and Genoa was a crucial event in the history of Medieval Europe. It brought about an expansion in the market which revived the economy of that time, firstly in Italy and then in the whole of Europe (Rosenberg and Birdzell, 1986). Until then, the western basin of the Mediterranean had been, as the Arab historian Ibn Khaldun observed, at the mercy of raids by the Muslims who, with the conquest of Sicily, had also virtually separated it from the Greek-Byzantine area (Pirenne, 2000 [1937], p. 156).

Political instability, emigration and underdeveloped trade due to the loss of supremacy over the Mediterranean were the underlying causes of the decline of the West in the early medieval period, while stability and expansion were responsible for its rebirth. Climatic factors may admittedly have led to emigration, but it must be recognised that other factors, intrinsic to western society, at the very least accentuated the effects of this population movement.

What we wish to stress here is that the crisis of the 1300s was admittedly triggered off by a worsening of the weather and above all by the plagues, but there is no doubt that the effects of this adverse change were accentuated by other factors. Among them was undoubtedly the decline in trade. This was caused by numerous factors: a decline in westward expansion; the downfall of the Mongol Empire, bringing an end to its monetary unity (although often inflated) but above all to the *pax mongolica* and its related trade (Lopez, 1966 [1962], p. 434); the collapse of the Crusader kingdoms; the decline of Byzantium and the expansion of the Turks; the Hundred Years War (Munro, 2001) and the general lack of peace in Europe and in Asia (Lopez, 1975 [1971], p. 211); less security, especially in the countryside; the collapse of important Italian banks; the bullion famine (and chiefly the scarcity of small change: Mayhew, 1974, Sargent and Velde, 1999, 2002). In the light of this, the climate may have acted as a trigger, but the ground had been prepared by political and economic forces.

Institutions are vital for providing the stability needed for the development of economic life (North, 1994 [1990], pp. 24-26, p. 72). Their weakness meant that they were unable to resist the force of events, and this led to a series of negative effects on economic activity and on the social structure.

The instability in the economic system, for example, was strongly felt in England, where peasants were subjected to heavy taxation between c. 1290 and the mid-1320s and where a shortage of coinage, along with other factors, created difficulties for the market-oriented peasant elite (Bailey, 1998).

The importance of economic and institutional factors, moreover, is also corroborated by the fact that the crisis was not felt everywhere: the Hanseatic League, for example, enjoyed considerable economic development in the second half of thirteenth century, sustained by the growth of its political influence, and this was in spite of the climatic adversities which continued in that period too.

However, as we have mentioned, there was another major debate about the effects of depopulation, caused by the Black Death (and other subsequent plagues) and by the above-mentioned bullion famine, on the economy of the late Middle Ages. The advocates of the neo-Malthusian-Ricardian theory argue that the negative effects of the bullion famine were compensated for by the drop in population, although recent research (Sussman,1998) finds no trace of it, except in France in the years before 1439. On the contrary, Day (1978), Miskimin (1964), Munro (2002), Mayhew (1974, 1987) and others have stated that the bullion famine was one of the causes of the economic stagnation in the years following the terrible epidemic. Munro (2002), however, has stated that the Black Death was followed by rampant inflation for about three decades). Munro (1979, 1994, 2001, 2002) and many others have also shown that credit reflected the supply of coinage rather than compensating for it. Several scholars have addressed the problem of the degree of monetisation of the medieval economy and of its relation with the quantity of money in circulation. Kohn (1999) has shown that the late Middle Ages can be divided into three periods of high availability of bullion separated by two periods of hiatus (the eleventh century and 1360-1460) and that this had a direct influence on commercial activity. The first period of scarcity, according to Kohn, was made harder by the need to transfer funds to the Holy Land. On the other hand, these difficulties, at least in the light of the analysis which we will present in the next sections of this article, were at least partly mitigated by the

expansion of the market brought about by the Crusades. Spufford has presented a well documented history of the use of money in the Middle Ages, emphasising, for the late Middle Ages, the increased importance of gold compared to silver, and the difficulties in trade due to the scarcity of money (1988, pp. 339-362), pointing out that in the mid-fifteenth century "the velocity of circulation was gravely reduced, to the great damage of both international trade and daily life". The problem of the bullion famine has also been studied by means of mathematical models, like those of Sargent and Velde (1999, 2002) who, unlike previous historians, claimed (2002, pp. 124-125) that this problem was principally due not to passive commerce with the East, but to an imperfect monetary system characterised by a currency the value of which, even for small change, was dependent on the fine metal it contained. A recent publication (Allen, 2001) has provided data and new estimates indicating that, at least in England in the mid-fourteenth century, there was actually a substantial decrease in the money in circulation, and particularly, but not exclusively, in small change.

2. The model

To highlight more clearly the factors that can determine monetary or financial crises in an economy partially based on commercial trade, we will use a simple mathematical model, describing the working of such an economic system.

Firstly, however, we shall present and discuss the general kind of function which we will use to construct it.

This is a production function:

$$Y = N^{\alpha} P^{\beta} K^{\gamma} \qquad \textbf{(1)}$$

Y represents the income of the economy, P the workers involved (which for the sake of simplicity we consider as coinciding with the population and therefore we call it P instead of L: we can consider in fact the average productivity of its various components), K the amount of capital available, while α, β and γ (with $\alpha + \beta + \gamma = 1$) are the

exponents quantifying the relative efficiency (and also the relative weight in the production process) of the three factors of production. Only K and P are to be regarded as variables, N being the "nature" factor, or the soil.

Equation **1**, in the case of a population that is still primitive, not organised and lacking technology (simple gatherers), can be re-written thus:

$$Y = N^{\alpha} \qquad \textbf{(2)}$$

with $\alpha = 1$. The fact that α is equal to 1 shows that production, in this case, is totally independent of human intervention.

To study the effect of the capital factor on the entity of the resources available for the population, it will be necessary to modify **1**, to permit it to cover the annual restoration cost of the capital itself. Therefore:

$$Y = N^{\alpha} P^{\beta} K^{\gamma} - r K \qquad \textbf{(3)}$$

where Y is now the production really available for the population P. We state beforehand that this analysis does not consider the distribution of income among the classes of the population. The capital restoration cost is only a net cost for the community considered as a whole and therefore it is not to be considered as an income for any class. It therefore represents the consumption of national capital during the production process. Assuming ω equal to the resources annually necessary for the survival of an average person (not to be confused with the minimum *per capita* revenue, historically determined, often bigger than ω), we can therefore determine the maximum size of the population P whose survival can be guaranteed by the system:

$$P = (N^{\alpha} P^{\beta} K^{\gamma} - r K) / \omega \qquad \textbf{(4)}$$

This relation can be re-written thus:

$$N^{\alpha} P^{\beta} K^{\gamma} - r K - \omega P = 0 \qquad \textbf{(5)}$$

Considering the first derivative of function **3**, with respect to K, equal to zero, we obtain the following system of equations:

$$\begin{aligned} & N^{\alpha} P^{\beta} K^{\gamma} - r K - \omega P = 0 \\ & \gamma N^{\alpha} P^{\beta} K^{\gamma-1} - r = 0 \end{aligned} \qquad \textbf{(6)}$$

By solving it, it is possible to calculate the optimal values of K and of P, for which Y is the maximum (and therefore also P). For the sake of simplicity we will assume $\omega = 1$ (this assumption also regards the already cited *Figure 1*).

The value that interests us is that of P (synthesis of calculi in note[5]), which is, besides the trivial value 0, the following:

$$P = N [(1 - \gamma)]^{(1-\gamma)/\alpha} (\gamma / r)^{\gamma/\alpha} \qquad \textbf{(7)}$$

It is important to note that the parameter β does not directly appear in **7**, even if its value influences $\alpha + \gamma$ anyway. Increases in β, in fact, accompanied by the simultaneous reduction of α and γ, always involve reductions in P (*Figure 1*): this means that in order to increase resources, what is decisive is the "lever effect" of the technical or natural means compared to the importance of the labour factor: the more important the latter is in the production process (or rather the poorer or more inefficient the other means are), the smaller the product will be.

[5] Our first step is to obtain K from the second equation of the system **6**:
$K^{\gamma-1} = r / (\gamma N^{\alpha} P^{\beta})$ and therefore $K = (\gamma N^{\alpha} P^{\beta} / r)^{1/(1-\gamma)}$.
Now we can insert it in the first equation:
$N^{\alpha} P^{\beta} (\gamma N^{\alpha} P^{\beta} / r)^{\gamma/(1-\gamma)} - r (\gamma N^{\alpha} P^{\beta} / r)^{1/(1-\gamma)} - P = 0$.
It can be rewritten
$P^{\beta/(1-\gamma)} / N^{\alpha} (\gamma N^{\alpha} / r)^{\gamma/(1-\gamma)} - r (\gamma N^{\alpha} / r)^{1/(1-\gamma)}] - P = 0$
or also
$P \{P^{\beta + \gamma/(1-\gamma)} [N^{\alpha} (\gamma N^{\alpha} / r)^{\gamma/(1-\gamma)} (1 - \gamma N^{\alpha})] - 1\} = 0$.
Therefore, besides the trivial solution $P = 0$, remembering that $\beta + \gamma = 1 - \alpha$, we obtain:
$P^{\alpha/(1-\gamma)} = N^{\alpha} (\gamma N^{\alpha} / r)^{\gamma/(1-\gamma)} (1 - \gamma N^{\alpha})$
and therefore
$P = [N^{\alpha} (\gamma N^{\alpha} / r)^{\gamma/(1-\gamma)} (1 - \gamma N^{\alpha})]^{(1-\gamma)/\alpha}$
It can be simplified:
$P = [N^{\alpha/(1-\gamma)} (1 - \gamma)]^{(1-\gamma)/\alpha} (\gamma / r)^{\gamma/\alpha}$
and finally
$P = N (1- \gamma)^{(1-\gamma)/\alpha} (\gamma / r)^{\gamma/\alpha}$.

This finding enables us to simplify the model, which will be very important later. In fact, because our intention is to determine the maximum population size, it will be possible to assume $\beta = 0$ from now on, and to concentrate on nature and capital factors.

Considering that if $\beta = 0$, then $\alpha = 1 - \gamma$, **7** can therefore also be rewritten thus:

$$P = N\alpha\,(\gamma / r)^{r/\alpha} \qquad \textbf{(8)}$$

However, before using the model to study the causes of the fourteenth century crisis and its persistence, its nature must be understood more clearly.

Figure 2 shows how the variable P evolves with opposite changes in γ and in α (with $\beta = 0$), for a stated r. We should specify that the rise in γ at the expense of α does not necessarily signify, in our model, a fall in the nature factor yield, but rather a relative rise in the importance of capital in the production process, accompanied therefore by a sort of progressive "disengagement" of the capital from the constraints of nature.

Evidently, as is also shown by *Figure 2* (explosive growth of P for α tending to zero, denoting a sort of "lever effect" of the capital), α cannot equal zero, because the limit of **7** (or **8**), for γ tending to 1, is infinity. The hypothesis of setting α to zero would not be realistic, as it would imply a complete disengagement not only from the limits imposed by nature in terms of space, but also from the resources themselves (raw materials and energy sources) provided by nature.

In any case, it is easy to note that before increasing, P has a slight initial reduction, the entity of which is directly influenced by r. The initial drop is in fact explained by the need to allocate a portion of labour to the creation and restoration of capital. For low values of γ, the latter is not adequately compensated by the revenue of the capital, which, given the function structure, is in any case necessary for carrying out production.

Figure 3, shows the variations of the maximum size of P, as r varies, given a certain value of γ, while Figure 4 shows all the combinations of γ and r consistent with a given maximum population. There exists, in fact, a direct relation between the values of γ and r consistent with a given

value of P (higher capital restoration costs must be compensated by a higher productivity of the capital itself).

It is obvious *(Figure 3)* that small variations in r are enough to cause far larger ones in the number of inhabitants the system can support.

As regards this quantity, which we can consider almost constant in the short term (like γ), it appears that the *range* of possible values is not very wide. Local variations are normally compensated for, or at least diluted, by their opposites, and significant global variations (rises) are possible for large systems only in consequence of severe upheavals like political disorder, wars or climatic changes[6] (factors that we do find in the fourteenth century).

At this point we can use the model as it is (and more specifically **7** or **8** obtained from it) to study the late Middle Ages, characterised, as we know, by a feudal economy which was gradually changing into a commercial economy, with an increasing territorial specialisation of labour and other production factors, thus triggering a process of economic growth almost without precedent.

Much has been written about the neutrality of money, strongly sustained by the neoclassical school (with Sidrauski - 1967 -, even super-neutrality). Here we do not want to dispute the fact that the employment of wealth in financial or real activities has no effect on the system. We think, however, that it is undeniable that money has an indispensable role in fostering commerce and, therefore, in the growth of the real economy, following specialisation of production factors. Moreover a drop in circulating money can hamper commerce because of a certain downward price rigidity. This is true at least of the first phases of the development of a mercantile society. It is possible to argue that, while it is true that the use of money is a consequence of labour specialisation, and that it is also promoted by the diversity of production in different geographic areas, the contrary is also true, i.e. that it is almost an essential

[6] It should also be pointed out that reductions in the efficiency of the nature factor (for example caused by negative climatic variations) might be taken into account by reducing N or, though less correctly, by increasing the value of the ratio α / γ (which indicates the need for more capital to obtain the same product and involves a reduction of the potentialities of the system).

condition for specialisation. The two activities, specialisation in the use of factors and use of money, are strongly correlated in an action-reaction process. The reason is simple: the use of monetary means drastically reduces transaction costs, making some specialised activities much more economical than they were previously. Money can therefore be considered a technological innovation which acts contemporaneously on the productivity of all three productive factors, due to their more specialised use.

The non-neutrality of money in the late medieval system is also credible in the long term, for two reasons:

1) while it is true that a growing scarcity of money could be offset by a fall in prices, it is also true that in an open system, as Europe was at that time, a scarcity of money - money being goods - actually implied a reduced purchasing power regarding the East. This was hard to rectify, given the scarce interest of the latter in western products (Day, 1987, Tangheroni, 1996, pp. 452-453), and it could imply a reduction in trade and therefore in the economy's specialisation level;

2) although prices could diminish, the initial impact on producers (possibly a return to self-consumption production) could have "inertia effects" on the economic system from which it was not always possible to recover rapidly.

We are not alone in thinking that money can play such a role. In the years when Keynes developed his famous theory, Bloch (1933) wrote that monetary phenomena are like a seismograph that not only registers earth tremors, but sometimes occasions them. But already, long before Keynes and Bloch, Malynes (1601) saw the use of money as a potent development factor, warning against the excesses and the deficiencies of its supply. Sir H. Steuart, in his "*Principles of Political Economy*" (1967 [1767], pp. 237-240) even tried "to deduce from the mere existence of money the emergence of a large variety of wants and commodities" (Cartelier, 2001, p. 288), and this idea has been taken up again in recent times by G. Simmel (1990).

Jean Cartelier (2001, pp. 285-299), following Simmel and comparing Steuart's thought to Smith's, emphasises the role of money as the

institutional factor underlying the specialisation and division of labour. This theory is the exact mirror image of the argument that the use of money is a mere consequence of this specialisation.

From our point of view, we wish to show that, in the end, the problem is of only relative importance: whether it is a cause or a consequence, the monetisation of an economic system is inseparably correlated to its specialisation. Commercial but not monetary activity may be possible, but the probability of it having the same intensity as monetary activity will certainly be far lower (Spufford -1988, p. 75-, in particular, wrote that "mint and market went together"). Of course, when we speak of monetisation we are not referring to the quantity of money in circulation, but rather to the portion of economic activity geared to a monetary income; in an economy which is still only partially monetary, this portion, however, is inevitably influenced by the amount of money available and also by its efficiency in carrying out transactions (which is also related to the existence of different-sized units).

It is therefore possible to represent the medieval economy in the period of the "Commercial Revolution" with a new production function that considers specialisation correlated to the monetisation of the system. It will take the form:

$$Y = \mu N^{\alpha} P^{\beta} K^{\gamma} - r K \qquad \textbf{(9)}$$

with $\mu \geq 1$.

Operating as for **3**, and remembering that we have assumed $\beta = 0$ and $\omega = 1$, we will obtain:

$$P = N \mu^{1/\alpha} \alpha (\gamma / r)^{\gamma/\alpha} \qquad \textbf{(10)}$$

The presence of the term μ quantifies the increase in income due to the specialisation of production factors - including geographical specialisation - brought by money. μ, in this sense, can be defined as a "coefficient of specialisation" of the economic system. In this way it makes otherwise non-homogeneous values comparable, such as capital and

differently specialised forms of labour. If $\mu = 1$ (absence or total inefficiency of money in the transactional role, for example from lack of confidence in it), **10** will coincide with **8**. It is very important to note that the effect of monetisation depends on the coefficient μ elevated to the exponent $1/\alpha$. This exponent grows if capital is more productive, i.e. if technology advances (and we know there was great technological progress in the late Middle Ages: therefore the monetisation effects became more and more important)[7]. Figure 5 shows the multiplicative effect of μ, for different values of itself, with opposite variations in γ and in α.

The value of μ, on the other hand, will probably depend on a series of factors that we need not examine closely here. We think it is sufficient to point out that, with reference to the quantity M of disposable (deflated) money, this will grow, though less and less quickly. If multiplied by the velocity of circulation, this quantity would give us only a fraction of the total national income, including production for self-consumption. However, the latter is unknown, and we think that deflated disposable money is a fair index of monetisation. We should not confuse this datum, expressed in pounds and deduced from past yearly figures (table 1), with Y, which is the total annual potential revenue calculated using the model, and expressed in terms of ω.

Thus an example of a rather simple formalisation might be the following:

$$\mu = (1 + M/\psi)^{\delta} \qquad \textbf{(11)}$$

[7] In addition, given money's distinctive function as a "specialisation catalyst", we can consider that γ, which measures *technological* level – though indirectly, through productivity – is also in some way influenced by μ (and conversely, technology can influence μ). In fact it is feasible to say that elevated values of γ are often consistent only with a high level of specialisation of the economy (in fact, specialisation fosters innovation: Smith, 1965 [1776], pp. 12-14). From this point of view a re-feudalisation of the system would, in the end, hamper the use of technology in production, if not actually reverting its use. Naturally we should not consider the μ–γ relation to be automatic. An increase in μ can support an increase in γ but certainly not determine it. As we know, in the various historical-geographical situations characterised by high trade levels, the technological and organisational progress has varied quite considerably. This has also been stressed by Ashtor (1989), who has, in fact, emphasised the importance of the migration of workers and the introduction of new raw materials, focusing on institutional factors ("the policy of the rulers").

where ψ is the price level and δ is a coefficient quantifying the effect of monetisation on the specialisation of the system, whose value depends principally on the size of the market (for larger states more money is required to achieve the same specialisation level, and so δ must be lower), but also on transport and payment techniques[8].

Certainly we do not expect particularly high values for μ. However, in a context like the medieval period which was characterised by a technological and organisational level not comparable with the current situation, and which could therefore be represented by a model with values of γ that are still not particularly high, the effects on the income of monetary factors (reduction of μ for scarcity of monetary means) and political factors (unproductive cost of the war, at least in the short-term: $+ \bar{r}$) might be more significant, in absolute terms, than the opposite effects caused by technological changes ($+ \gamma$); these, moreover, seem to some scholars to have slowed down during the fourteenth century (Ashtor, 1989).

3. Application to England

The model that we have constructed may provide a representation of the economic events of that time and it may therefore enable us to trace some of the possible causes of the slow recovery. For this purpose we will limit our study to England alone. We know that it is not a good illustration of the European economic situation of the late Middle Ages (in fact England, unlike other countries, had the features of a monopolistic state in a Weberian sense) but we have sufficient data to analyse the case of England.

The main problem we have to solve in order to use the model is that of assigning reliable values to the parameters. Given that the scant data available are uncertain, and also given that the model we propose is new, we will inevitably resort largely to conjectures. The present study, therefore, is to be considered only as an attempt to adapt the model to the historical situation which is the subject of this paper.

As we have already done for the graphics in Figures 1, 2, 3 and 4, we

[8] On the relationship between transport and economic development, see the contribution of Salvemini (1973), who reports on the 1973 annual meeting organised by the "Istituto Internazionale di Storia Economica Francesco Datini".

will suppose that ω is equal to 1. To obtain the value of N we will use n.**2**. Having assumed $\omega = 1$, we will have $P = N$, with N being, therefore, the number of inhabitants that the territory can sustain without intervention of capital. If we consider the estimates made for early Neolithic England, with a population density between 0.1 and 1 inhabitant per square kilometre (Hassan, 1981, Livi Bacci, 2002 [1998], p. 38) and overlooking the fact that in that period rudimentary hunting tools were used and are considered a primitive form of capital, we can assume a value for N of about 65,000, the surface area of England being about 130,000 square kilometres. As far as r is concerned, because we have no data related to the restoration times of the capital invested, we will assume a value equal to 0.1. This assumption may seem rash, but there are two reasons for this decision. Firstly the problem is not particularly important, given that here it is more important to study the relative variations of certain parameters than their absolute values. Assigning r a different value, as we have seen before *(Figure 4)*, will therefore make it necessary only to calibrate the other parameter γ to represent a similar phenomenon. On the other hand, the value that we will give r (0.1) is a likely value considering that for the late middle ages we have an estimate about the life cycle of ships (about five years, if we make a weighted average), supplied by Melis (1985: table 2). If, for the sake of simplicity, we ignore the repairs made on the ships, we can then estimate a restoration rate of 0.2 for the fleet of that time. Considering that marine investment was exposed to higher risks and to greater wear and tear, for r (which regards the entire economy) we can assume a value between 0 and 0.2, namely 0.1.

The last problem regards the parameters γ and δ (α is simply equal to $1 - \gamma$).

To assign reliable values to γ we will compare the data available for England related to different historical periods. There are many conflicting estimates of the English population before the Great Famine and the Black Death[9]. Britnell (1991) has considered a population of six million inhabitants in the first half of the fourteenth century in England. If

[9] We can cite Nightingale (1996, pp. 89-106), who estimates a population of not over 4.5 million, and Hallam (1988, pp. 508-593), who calculates 6.74 million for 1317 and 7.2 million for 1290.

compared with the estimates by Russell of 3.7 million inhabitants (1979 [1973], p. 25), this figure seems excessive but it has been put forward again in recent times by other scholars (Mayhew, 1995, Allen, 2001) from whose articles we have obtained some monetary data used in the present work. At any rate, by examining Russell's estimates, we observe that, for the years around 500 and 650, when money was hardly used, the population of the British Isles was about 500,000 inhabitants. If we suppose that the population of England at that time, as in the late Middle Ages (Russell, 1979 [1973], p. 25) and in the present day, fluctuated from 60 % to 80 % of that of the British Isles, we can estimate that in the years 500 and 650 England was inhabited by 350,000 people (70% of 500,000). We can assume (very approximately) that this population with its stationary nature was at its peak. We know (Spufford, 1988, p. 379) that it was sustained by a non-monetary economy ($\mu = 1$) and we can assume that it was characterised by a higher value of r too. In fact there was no peace in that period and therefore r might be higher than during the Roman period or during the Commercial Revolution (for both these periods we assume $r = 0.1$). Therefore, we will consider the situation in England in Roman times. We will assume that the fact that the population was larger in that period than in the early Middle Ages (approximately 1,000,000 inhabitants, based on the estimate quoted by Reinhard, Armengand and Dupaquier, 1971 [1968], p. 65, of a maximum population of 1,500,000 inhabitants for the British Isles) was due only to the better organisation (monetisation) of the economy, since the technology was not noticeably different, and to the lower value of r, because of peace. Supposing, for the sake of simplicity, that the successive reduction in the population was due, in equal parts, to the two causes ($+ r$ and $- \mu$)[10], then by using **10** we can complete the estimates of the values of μ, γ and r for the two periods (the Roman period and the early middle ages).

We thus obtain (all the data and the results are in Table 3): for the Roman era, $\mu = 1.150067$, $\gamma = 0.6442622$ ($\alpha = 0.3557378$), $r = 0.1$; for

[10] The population actually decreased partly because of the worsening of climatic conditions (Lopez, 1966 [1962], pp. 34-36), but it is practically impossible to assess their effect.

the early Middle Ages, $\mu = 1$, $\gamma = 0.6442622$, $r = 0.1437132$ (all the values of these parameters, now and later, are rounded to the 7th decimal point).

We can now consider the fourteenth century. If we assume (with a good measure of approximation) that the economy of the 1300s was monetised like that of Roman Britain, we can fix the same value of μ. The difference in population (which we might presume to be near to the maximum allowed by the economic system of that time) will therefore depend mostly on the differences in the technological level, besides the differences in the productive system, given the peripheral position of the province within the Roman Empire and its scarce political weight, which are all characteristics that influence cultivation techniques. Thus, for a population of 6,000,000 inhabitants we have: $\gamma = 0.7279962$ ($\alpha = 0.2720038$). If, instead, we use Russell's estimates (1979 [1973], p. 25), which give a population of 3.7 million, we obtain: $\gamma = 0.7099433$ ($\alpha = 0.2900567$).

We can now consider the amount of money in circulation in the years before the Black Death. The estimates made by Allen (2001) are more up-to-date for 1310, 1319 and 1331 and we will therefore concentrate on those years. It is feasible that the money in circulation in 1310 and 1331, almost the same amount, was approximately the amount above which any greater supply would only have generated inflation, and that there was not a high degree of specialisation. In fact the years immediately before 1320, when the amount of money was greater, saw substantial price rises (Munro, 2002: *Table 1)*, which are obviously related to the climatic adversities of those years. In the hope of not being too far from the truth, we could therefore estimate the value of δ for 1310, substituting the values of M (for which we will consider an average of £1,700,000) and ψ (related to 1 and not to 100) in **11**.

We will thus obtain the value of δ, i.e. 0.1494829, which we can assume as constant for the period under examination.

This allows us to obtain values of some reliability for μ related to the second half of the fourteenth century and the first half of the fifteenth, basing our calculations once again on the data furnished by Allen's estimates *(Table 4)* and adding them to **11**, together with the price levels *(Tables 1 and 5)*. As is shown by *Table 5*, μ should have values between 1.084324

and 1.105893. Naturally we have considered γ stable for this period, both for the sake of simplicity and because a technological advancement may really have been made uninfluential by worsening in climatic conditions. As we have specified (note 4), climatic deterioration tends to decrease the value of N, but this phenomenon can also be described, though less correctly from a formal viewpoint, by an increase in the α / γ ratio with a lowering of the productive potentialities of the system. The calculations, carried out once again by substituting the values of μ just obtained in **10**, show that the biggest population P consistent with them, in the period after the Black Death, was reduced by over half a million inhabitants for $\gamma = 0.7099433$ (population pre-Black Death 3.7 million inhabitants) and about 1 million inhabitants for $\gamma = 0.7279962$ (population pre-Black Death 6 million); this was without considering variations in r (*Table 5* and *Figure 6*). The data quoted in columns 5 and 7 of Table 5 (and set out in *Figure 6*: middle line) show that the effects of a variation of r were even greater: in fact even a mere increase of r from 0.10 to 0.11 (plausible in the period of the Hundred Years' War, even though the war was fought in France) could considerably reduce P. This was because r was more important in an economy that was more advanced than the Roman one, and therefore characterised by a higher value of γ.

It is important to note, moreover, that we have not considered that in the years after the Black Death money was scarce, especially money of small denomination (Sargent and Welde, 2002, pp.131-135) necessary for retail trade, and so our analysis of the effects of the availability of money probably underestimates the effects of demonetisation.

Therefore, even if the effects of the reduction of M have been at least partially absorbed by an increase in direct trade (this would involve a less significant influence of the reduction of M on μ), it cannot be denied that these, together with the war, strongly influenced the economic trend of that time. If we consider the increase in r caused by the war and by the consequent uncertainty that reigned in England like most of Western Europe (Munro, 2002), and also that income per head was practically uncompressible, being already near the pure subsistence level ω, we can understand why we should not expect a rapid recovery after the Black Death but rather stagnation.

It is evident that the decline in population density following the Black Death and the successive plagues influenced production not only directly, in accordance with Postan's theory, but also through its effect on μ: people did have the opportunity to cultivate better lands, but were driven towards an economy of self-sufficiency, which was less specialised, less monetary and commercial, because of the twin effects deriving from an decrease in M and an increase in r (on account of the cost of the war). The decreased money supply, then, far from being a factor that redressed the balance, can be regarded, together with the political and social instability, as one of the principal reasons that prevented the English economy from making a quick recovery.

4. Conclusions

Even though the work on the model in the previous section centres around the case of England, it highlights the importance of the political and monetary factors and the importance of harsh (and unproductive) taxation in an economy characterised by a vigorous growth in trade. Naturally this does not mean ignoring the climatic factor, which could have a significant influence on a population that was still largely engaged in subsistence agriculture, and the importance of plagues in determining the crisis. It can, however, be concluded that a significant influence may have come from the political situation which, in turn, was influenced by monetary factors. This situation affected both the commercial and the agricultural sector (either providing protection or not providing adequate protection against internal and external enemies), and played, as it still does, a role of the greatest importance in optimising the value of economic parameters.

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Appendix

Development and Crisis in the Late Middle Ages: the Role of Trade

FIGURE 1. Relation between potential population and β

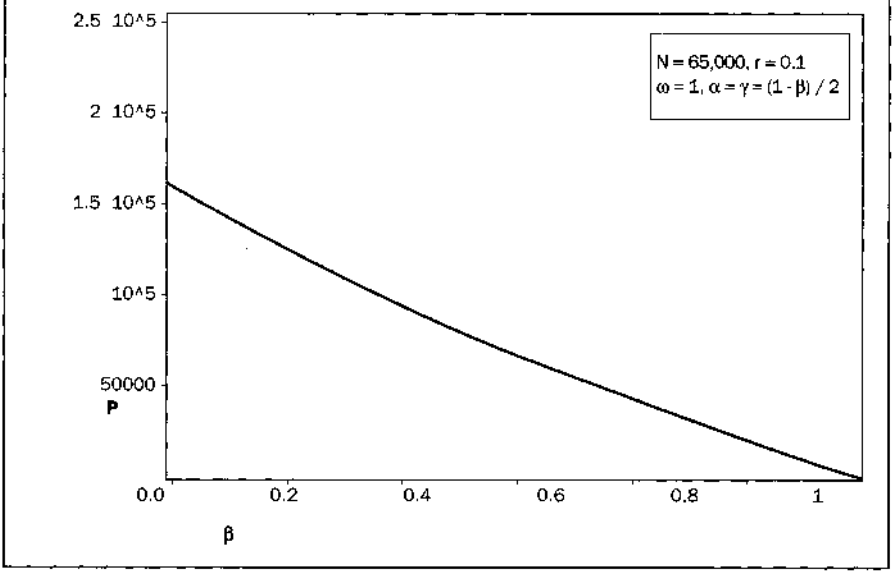


FIGURE 2. Relation between potential population and γ

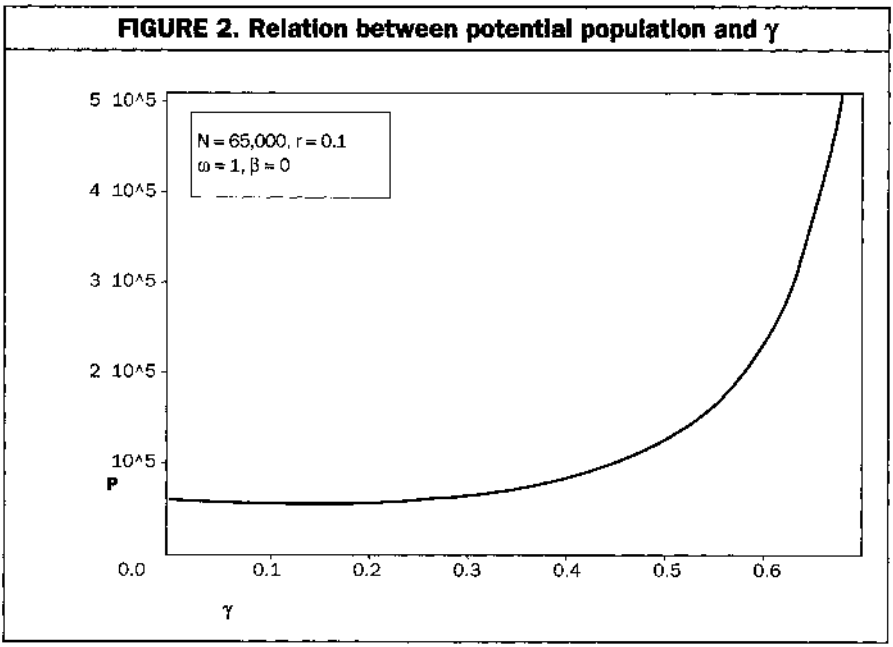


FIGURE 3. Relation between potential population and r

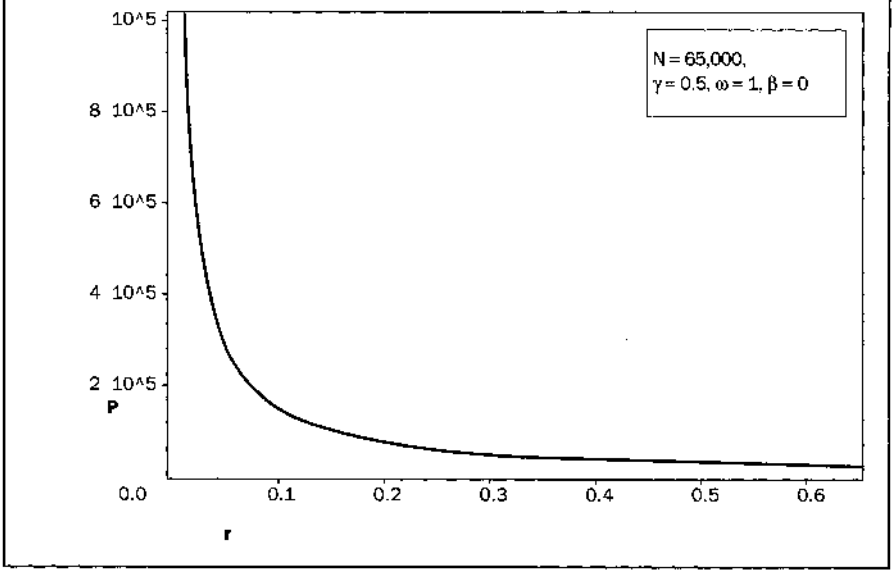


FIGURE 4. Compatible combinations of values of r and γ with a certain potential population

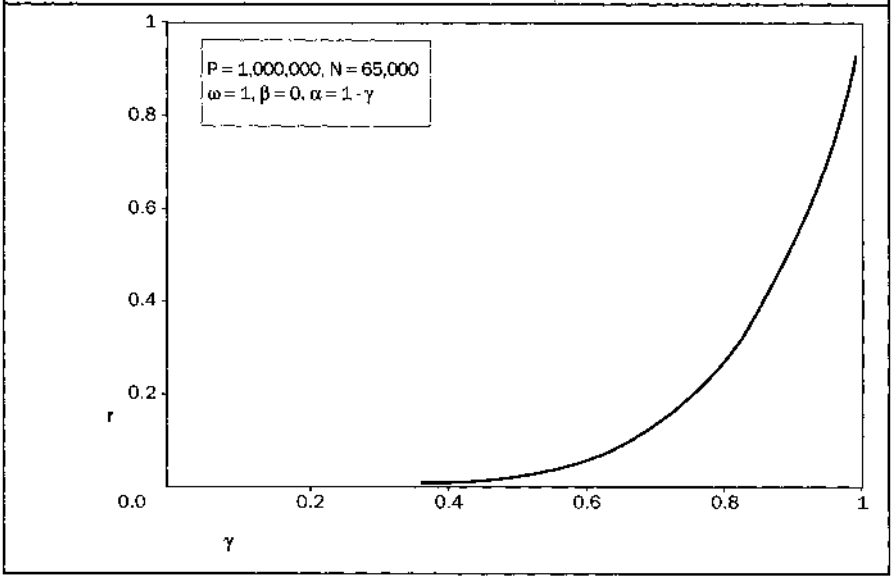


FIGURE 5. Multiplicative effect of γ for different values of μ

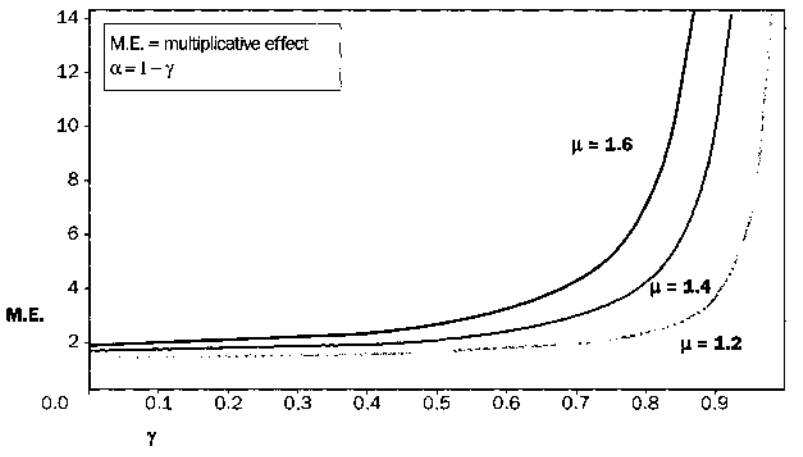
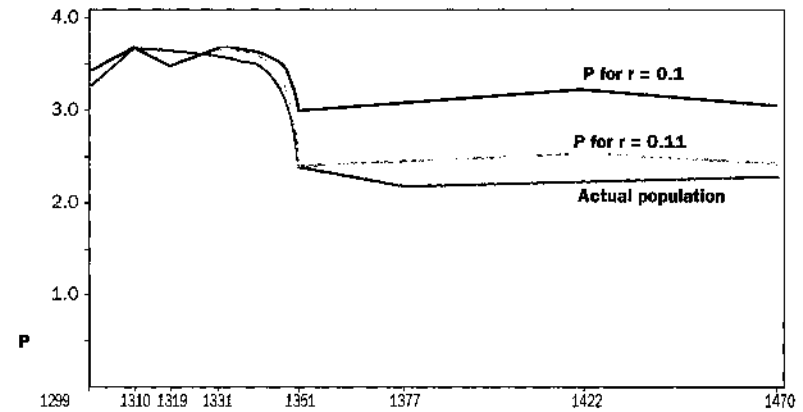


FIGURE 6. Actual English population compared with the one foreseen by the model, 1299-1470



The data about the actual population are essentially derived from Reinhard, Armengaud and Dupaquier (1971 [1968]), pp. 107, 140, 143 and Mayhew, 1995. For the whole period the value of γ is assumed constant and equal to that calculated for 1311, assuming that the population was at its peak at 3.7 million. The other lines are constructed using the model (relations 10 and 11), with $r = 0.1$ and, from 1351 to 1470, also for $r = 0.11$. P and the actual population are expressed in millions. For calculations about 1331 we have overlooked the little amount of gold coin.

TABLE 1. English mint outputs, 1296-1475

Years	Silver in kg	Silver in £ sterling	Gold kg	Gold £ sterling	Total value in £ sterling	PB & HC Price Index
1296-00	12,071.417	37,761.545			37,761.545	102.34
1301-05	16,017.465	50,105.484			50,105.484	92.35
1306-10	40,226.553	125,835.827			125,835.827	109.81
1311-15	10,706.712	33,492.502			33,492.502	115.33
1316-20	7,275.676	22,759.610			22,759.610	161.91
1321-25	1,780.107	5,568.492			5,568.492	137.97
1326-30	121.857	381.190			381.190	111.07
1331-35	209.056	665.131			665.131	114.12
1336-40	429.488	1,551.599			1,551.599	94.32
1341-45	5,077.456	17,710.473	240.011	9,859.484	27,569.958	90.06
1346-50	1,991.051	7,090.874	675.837	27,123.297	34,214.171	102.70
1351-55	17,442.905	67,245.275	1,939.777	83,567.731	150,813.007	132.18
1356-60	4,423.016	17,081.461	1,726.695	74,406.844	91,488.305	129.46
1361-65	1,630.811	6,298.107	2,415.242	104,077.756	110,375.864	146.64
1366-70	293.822	1,134.727	1,729.027	74,507.352	75,642.079	146.10
1371-75	316.966	1,224.108	802.608	34,586.019	35,810.127	135.26
1376-80	356.898	1,378.322	235.330	10,140.847	11,519.169	110.62
1381-85	317.412	1,225.829	161.835	6,973.804	8,199.633	112.90
1386-90	247.514	955.887	504.811	21,753.331	22,709.218	102.53
1391-95	193.489	747.245	626.546	26,999.152	27,746.397	106.33
1396-00	175.596	678.143	391.143	16,855.142	17,533.285	110.84
1401-05	66.344	256.216	168.671	7,268.390	7,524.606	114.84
1406-10	10.592	40.907	69.005	2,973.568	3,014.475	111.23
1411-15	967.484	4,483.340	1,870.669	89,519.896	94,003.236	108.11
1416-20	837.763	3,882.476	1,035.150	49,563.076	53,445.552	113.40
1421-25	3,186.020	14,765.093	2,557.314	122,444.369	137,209.462	101.48
1426-30	6,858.608	31,785.107	599.478	28,703.069	60,488.176	112.27
1431-35	8,059.545	37,350.656	220.785	10,571.183	47,921.839	108.48
1436-40	977.025	4,527.863	132.274	6,333.298	10,861.161	122.01
1441-45	130.700	605.707	90.778	4,346.467	4,952.174	92.53
1446-50	517.373	2,397.681	64.336	3,080.422	5,478.103	100.90
1451-55	1,460.637	6,769.085	63.526	3,041.629	9,810.714	100.25
1456-60	1,415.094	6,558.024	26.719	1,279.288	7,837.312	97.06
1461-65	3,432.915	18,067.349	488.118	29,731.331	47,798.679	102.73
1466-70	5,168.090	29,938.348	1,288.157	83,263.992	113,202.339	106.75
1471-75	2,422.654	14,034.247	538.669	34,818.552	48,852.799	97.76

English Mint Outputs, 1296-00 to 1471-75, in kilograms of pure silver and gold and in pound- sterling values with the Phelps Brown & Hopkins Composite Price Index: 1451-75 = 100 in quinquennial means (source: Munro, 2002, itself derived from Challis, 1992, Munro, 1973, 1981, 1983, 1984, 1991)

TABLE 2. Ships and their average duration, 1383-1411

"Botti" (or barrels: a)	Ships (b)	Duration in years (c)	Total burden TB = a x b	TB x c
1,200	15	6.60	18,000	118,800
1,100	2	9.00	2,200	19,800
1,000	22	6.14	22,000	135,080
900	18	5.78	16,200	93,636
800	33	5.79	26,400	152,856
700	37	4.62	25,900	119,658
600	48	3.92	28,800	112,896
500	59	4.56	29,500	134,520
400	61	3.13	24,400	76,372
300	71	3.20	21,300	68,160
200	48	2.69	9,600	25,824
100	17	3.94	1,700	6,698
			226,000	1,064,300

The data of columns 1, 2 and 3 are derived from Melis (1985), pp. 14 and 33-34 and concern the years 1383-1411. The last figure in column 3 is slightly rectified. The total of column 5, divided by the total of column 4, gives the average duration of a *botte*, a medieval burden measure, that enables us to estimate ship maintenance and repair costs.

TABLE 3. Actual population of Britain in different times and estimate of parameters values (particularly of δ)

	Early Neolithic	Roman Britain	500-650	1310
Actual population (assumed, for these years, equal to P, i. e its maximum level)	c. 65,000	c. 1,000,000	c. 350,000	from 3,700,000 to 6,000,000
r	-	0.1	0.1437132	0.1
γ	0	0.6442622	0.6442622	From 0.7099433 to 0.7279962
μ	1	1.150067	1	1.150067
δ				0.1494829

(for $\omega = 1$ and $\beta = 0$); r and μ are assumed to be the same for the Roman period and the Late Middle Ages, γ for the Roman period and the Early Middle Ages; for δ, the only important figure is that of 1310, i.e the result of calculations.

TABLE 4. Estimate of English currency, 1299-1470

Years	Money stock	Average money stock
1299	1.1-1.4	1.25
1310	1.5-1.9	1.7
1319	1.9-2.3	2.1
1331	1.5-2.0 (+ gold)	c. 1.75
1351	0.8-1.1	0.95
1422	0.95-1.0	0.975
1470	0.75-0.95	0.85

The estimates of the English currency (£ million) are derived from Allen (2001).

TABLE 5. Actual English population compared with the one foreseen by the model, 1351-1470

Years	ψ = Price level / 100	μ	P (If P of 1310 = 3.7 million) r = 0.10	P (If P of 1310 = 3.7 million) r = 0.11	P (If P of 1310 = 6 million) r = 0.10	P (If P of 1310 = 6 million) r = 0.11	Actual population
1351	1.3218	1.084324	3,020,420	2,391,963	4,832,443	3,744,403	c. 2.4 - 2.2
1422	1.0148	1.105893	3,232,647	2,560,032	5,195,352	4,025,602	million
1470	1.0675	1.091499	3,089,884	2,446,974	4,951,046	3,836,302	

(for $\omega = 1$, $\beta = 0$, $\delta = 0.1494829$)

The data about the actual population are essentially derived from Reinhard, Armengaud, Dupaquier, 1971 [1968], pp. 140, 143, and Mayhew, 1995. Figure 6 shows these yearly tables from 1299 to 1470 and is more precise regarding the actual population.

problems